

①

Quiz:

Solution:

$$1:2:1, 2:1:1, 2:0:2$$

P	E
A	E

P	P
A	E

P	P
A	A

Let  $x, y$  &  $z$  be the cost/kg Pak, Egyptian, American Cotton respectively, then according to the given conditions:

$$\left. \begin{aligned} \frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z &= 40 \\ \frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z &= 50 \\ \frac{2}{4}x + \frac{2}{4}z &= 60 \end{aligned} \right\} \Rightarrow A$$

$$\left. \begin{aligned} 1x + 2y + 1z &= 160 \\ 2x + 1y + 1z &= 200 \end{aligned} \right\} \rightarrow (B)$$

$$1x + 1z = 120$$

(2)

In matrix form, we can write as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$AX = B$$

$$\Rightarrow A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix} = A_2 \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

First  $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$  Expand by  $R_1$ .

$$= 1 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(1 \times 1 - 1 \times 0) - 2(2 \times 1 - 1 \times 1) + 1(2 \times 0 - 1 \times 1)$$

$$= -2$$

(3)

Now.

$$|A1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix} \text{ Expand by } R_1$$

$$= 160 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1 \begin{vmatrix} 200 & 1 \\ 120 & 0 \end{vmatrix}$$

$$|A1| = -120.$$

Similarly

$$|A2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix} \text{ Expand by } R_1.$$

$$= 1 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} - 160 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix}$$

$$|A2| = -40.$$

$$|A3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix} \text{ Expand by } R_1.$$

$$= 1 \begin{vmatrix} 1 & 200 \\ 0 & 120 \end{vmatrix} - 2 \begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix} + 160 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

(9)

$$= 1(120-0) - 2(240-200) + 160(0-1)$$

$$|A_3| = -120$$

now according to Cramer's Rule -

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = 60$$

$$(x, y, z) = (60, 20, 60)$$

Pakistan = 60

Egypton = 20

American = 60

