PROB & STATISTICS
FINAL'S

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Bs CS 4^{th}

Q1.

[redacted] → Let,

A = { The sum is 7 }
B = { The sum is even }
C = { The sum is greater than 8 }
D = { The 2 dice had same outcome }

Thus,

A = { (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) }

B = { (1,1), (1,3), (1,5), (2,4), (2,2), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6) }

C = { (3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6) }

D = { (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) }

So,

$A \cap B = 0$
$A \cap C = 0$
$A \cap D = 0$

$P(A) = \frac{6}{36}$, $P(B) = \frac{18}{36}$, $P(C) = \frac{10}{36}$,

$P(D) = \frac{6}{36}$

Thus,

$P(A \cap B) = 0$
$P(A \cap C) = 0$
$P(A \cap D) = 0$

Then, $P(A/B) = 0$

$P(A/C) = \frac{P(A \cap C)}{P(B)}$

$= 0$

$P(A/D) = 0$ As well.

(Q NO 2)

→

When 2 dices are rolled, there are a total of 36 possible different combinations made, out of which 15/36 for each side with a sum of 30/36.
The 15 possible combinations less than 7 are.

(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1).

→ The 15/36 for each side with a sum of 30/36
→ This leaves us with a 6/36 = 1/6 probability for a sum of 7

$$6/36 = 1/6.$$

→ Which gives us total of 21 possibilities for less or equal to 7, 15 for more than 7 and.

(Q NO 3)

→

$P = \frac{2}{3}$,

$n = 8$

And,

$q = 1 - P$

so,

$1 - 2/3$, $q = \frac{1}{3}$

Now,

1. EXACTLY 4 GAMES , $P(x = 4)$:

→ $\binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$

$= \frac{1120}{6561}$

$= 0.1707$.

2. ATLEAST 4 GAMES $P(x \geq 4)$

$\rightarrow \quad 1 - P(x < 4)$

So,

$$= 1 - \sum_{x=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= 1 - \left[\left(\frac{1}{3}\right) + 8\left(\frac{2}{3}\right)\left(\frac{1}{7}\right)^{7} + 28\left(\frac{2}{3}\right)^{2}\left(\frac{1}{3}\right)^{6} \right.$$

$$\left. + 56\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} \right]$$

$$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561} = 0.912.$$

3. FROM 3 to 6 GAMES $(P(3 \le x \le 6)$:

$$\rightarrow \sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-4}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{4} + \binom{8}{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{3}$$
$$+ \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{3^8}\left[56+140+224+224\right]$$

$$= \frac{8 \cdot 644}{6561}$$

$$= \frac{5152}{6561} \Rightarrow 0.7852$$

(Q NO 4)

ANS :

→ By applying the law of probability for $A \cap B$,

$$P(A \cap B) = \sum_{i=1}^{M} P(A \cap B / C_i)\ P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{M} P(A/C_i)\ P(B/C_i)\ P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{M} P(A/C_i)\ P(B)\ P(C_i)$$

(As B is independent of C_i)

$$P(A \cap B) = P(B) \sum_{i=1}^{M} P(A/C_i)\ P(C_i)$$

$$P(A \cap B) = P(B)\ P(A)$$

(Hence proved the law of total probability)

A&B are independant.

(Q NO 5)

→ BI-NOMIAL DISTRIBUTION & ITS MEAN AND VARIANCE:

→ The prob-func for a binomial variable is,

$$b(x n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

→ If x is a random variable with this property. probability distribution,

$$E(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

→ As $x = 0$ vanishes, Let $y = x-1$ and $m = n-1$

$x = y+1$ & $n = m+1$

$$\Sigma(x) = \sum_{y=0}^{m} \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y}$$

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$$= (m+1)p \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

Set $a = p$ and $b = 1-p$

$$\sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} =$$

$$\sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^y b^{m-y} = (a+b)^m = (p+1-p)^m = 1$$

So,

$$[E(X) = np]$$

Similarly,

$$E(X(X-1)) = \sum_{x=0}^{n} x(x-1)\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{n-x}$$

$$= n(n-1)p^2 (p+(1)-p))^m$$

$$= n(n-1)p^2$$

So variance of X:

$$E(X^2) - E(X)^2 = E(X(X-1)) + E(X) - E(X)^2 =$$

$$n(n-1)p^2 + np - (np)^2 =$$

$$[np(1-p)]$$

(Q NO 6)

→ Bi-NOMIAL FREQUENCY DISTRIBUTION:

→ If the binomial probability distribution is multiplied by N, the number of experiments or sets the resulting distribution is known as bi-nomial frequency distribution.

FORMULA :

$$N \binom{n}{x} p^x q^{n-x}$$

→ Bi-NOMIAL DISTRIBUTION:

→ The bi-nomial distribution can be thought of as simply the probability of a success or failure outcome in an experiment or survey that is repeated multiple times.

FORMULA :

$$P(x-x)\ f(x) = {}^nC_x\ p^x\ q^{n-x}$$

(Q NO 7)

→ Find mean & standard dev of several data set ...... Round to the nearest tenth.

→ **DATA SET A :**

The co-efficient of variation,

$$CV = \frac{\sigma}{\mu} \times 100$$

So, $CV = \frac{3}{45} \times 100 = 6.7$.

→ **SET B :**

$CV = \frac{11}{60} \times 100$, $\Rightarrow$ 18.3

→ **SET C :**

$CV = \frac{5}{50} \times 100 \rightarrow 10$.

→ SET D :

$$CV = \frac{15}{25} \times 100 \Rightarrow 60.$$