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Date:

(1)

Question 1

Answer

The same space S for this experiment is

$S = \{ (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2)$
$(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3)$
$(3,4),(3,5)(3,6),(4,1)(4,2),(4,3),(4,4),(4,5)$
$(4,6),(5,1)(5,2),(5,3)(5,4)(5,5),(5,6),(6,1)$
$(6,2)(6,3),(6,4),(6,5),(6,6) \}$

Let A = { the sum is 7 }
B = { the sum is odd }
C = { the sum is greater then 6 }
and
D = { the two die lad the same outcome }
then

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$B = \{(1,2),(1,4),(1,6)(2,1),(2,3),(2,5),(3,2) \ldots\ldots (6,5)\}$

$C = \{(1,6),(2,5),(2,6),(3,4),(3,5),(3,6) \ldots\ldots (6,6)\}$

$D = \{(1,1)(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$A \cap B = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$A \cap C = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$A \cap D = \phi$

$P(A) = 6/36$, $P(B) = 18/36$, $P(C) = 21/36$

$P(D) = 6/36$

$P(A \cap B) = 6/36$, $P(A \cap C) = 6/36$ and $P(A \cap D) = 0$

Hence

$$P(A/B) = \frac{P(A \cap B)}{P(C)} = \frac{6}{36} \times \frac{36}{18} = 1/3$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{6}{36} \times \frac{36}{21} = 2/7$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = 0 \times \frac{36}{6} = 0$$

Question 2

Answer

As there are 6 numbers on one die and 6 on the other, the total number of ways the dice can fall is 36.

Let (1,4) represent rolling a 1 with the first die and a 4 with the second and so on.

A sum of 2 can be obtained in only 1 way -from (1,1)

A sum of 3 can be obtained in 2 ways - (1,2) and (2,1)

A sum of 4 can be obtained in 3 ways - (1,3) , (2,2) and (3,1)

A sum of 5 can be obtained in 4 ways - (1,4) , (2,3) , (3,2) and (4,1)

A sum of 6 can be obtained in 5 ways - (1,5) , (2,4) , (3,3) , (4,2) and (5,1)

A sum of 7 can be obtained in 6 ways - (1,6) , (2,5) , (3,4) , (4,3), (5,2) and (6,1)

A sum of 8 can be obtained in 5 ways - (2,6) , (3,5) , (4,4) , (5,3) , and (6,2)

A sum of 9 can be obtained in 4 ways - (3,6) , (4,5) , (5,4) and (6,3)

A sum of 10 can be obtained in 3 ways - (4,6) , (5,5) and (6,4)

A sum of 11 can be obtained in 2 ways - (5,6) and (6,5)

A sum of 12 can be obtained in 1 way - (6,6)

A sum greater than 7 occurs when the total is 8,9,10,11 or 12.

From the list above, this can happen in 5 + 4 + 3 + 2 + 1 = 15 ways

The probability of obtaining a sum greater than 7 is 15/36 = 5/12 or approx 0.42.

Date:

Q 3:

Answer

Give that $p = 2/3$ $n = 8$

$q = 1 - p$

$= 1 - 2/3$

$q = 1/3$

Let "x" denote the number of games won by A, then

1) $P(x = 4)$

$= \binom{8}{4} (2/3)^4 (1/3)^4$

$= \frac{1120}{6561}$

$= \boxed{0.1707}$

(ii) $P(x \geq 4)$

$1 - P(x < 4)$

$= 1 - \sum^{3} \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$

$= 1 - \left[(1/3)^8 + 8\left(\frac{2}{3}\right)(1/3)^7 + 28(2/3)^2(1/3)^6 + 56(2/3)^3 (1/3)^5\right]$

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$$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$\boxed{= 0.9121}$$

(iii) $P(x \geq 6)$

$$\sum_{x=6}^{8} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2} + \binom{8}{7}\left(\frac{2}{3}\right)^{7}\left(\frac{1}{3}\right) + \binom{8}{8}\left(\frac{2}{3}\right)^{8}\left(\frac{1}{3}\right)^{0}$$

$$= \frac{64}{6561}\left(28 + 16 + 4\right)$$

$$= \frac{64 \times 48}{6561}$$

$$= \frac{1024}{6561} \Rightarrow \boxed{0.4682}$$

Date:

iv) $P(3 \le x \le 6)$

$$\sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{4} + \binom{8}{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{3}$$

$$+ \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{(3)^8}\left[56 + 140 + 224 + 224\right]$$

$$= \frac{8 \times 644}{6561}$$

$$= \frac{5152}{6561}$$

$$= \boxed{0.7852}$$

Darsi Notes

Date:

(Q 4)

(Answer)

Proof:-

Since the C_i's form a partition of the sample space we can apply the Law of total probability for $A \cap B$.

$$P(A \cap B) = \sum_{i=1}^{m} P(A \cap B / C_i)\, P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i)\, P(B/C_i)\, P(C_i)$$

:- (A and B are Conditionally independent)

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i)\, P(B)\, P(C_i)$$

:- (B is independent of all C_i's)

$$P(A \cap B) = P(B) \sum_{i=1}^{m} P(A/C_i)\, P(C_i)$$

$$P(A \cap B) = P(B)\, P(A)$$

:- Law of total probability

Hence A and B are independent

Darsi Notes

Question 5

Answer:

Mean and Variance of Binomial Random Variables

The probability function for a binomial random variable is

b(x;n,p) = n x􀀀px(1 − p)n−x

This is the probability of having x successes in a series of n independent trials when the probability of success in any one of the trials is p. If X is a random variable with this probability distribution,

$$\begin{aligned}E(X) &= \sum_{x=0}^{n} x\binom{n}{x}p^x(1-p)^{n-x}\\ &= \sum_{x=0}^{n} x\tfrac{n!}{x!(n-x)!}p^x(1-p)^{n-x}\\ &= \sum_{x=1}^{n} \tfrac{n!}{(x-1)!(n-x)!}p^x(1-p)^{n-x}\end{aligned}$$

since the x = 0 term vanishes. Let y = x−1 and m = n−1. Subbing x = y+1 and n = m+1

into the last sum (and using the fact that the limits x = 1 and x = n correspond to y = 0

and y = n − 1 = m, respectively)

$$\begin{aligned}E(X) &= \sum_{y=0}^{m} \tfrac{(m+1)!}{y!(m-y)!}p^{y+1}(1-p)^{m-y}\\ &= (m+1)p\sum_{y=0}^{m} \tfrac{m!}{y!(m-y)!}p^y(1-p)^{m-y}\\ &= np\sum_{y=0}^{m} \tfrac{m!}{y!(m-y)!}p^y(1-p)^{m-y}\end{aligned}$$

The binomial theorem says that

$$(a+b)^m = \sum_{y=0}^{m} \tfrac{m!}{y!(m-y)!}a^y b^{m-y}$$

Setting a = p and b = 1 − p

$$\sum_{y=0}^{m} \tfrac{m!}{y!(m-y)!} p^y (1-p)^{m-y} = \sum_{y=0}^{m} \tfrac{m!}{y!(m-y)!} a^y b^{m-y} = (a+b)^m = (p+1-p)^m = 1$$

so that

$E(X) = np$

Similarly, but this time using $y = x - 2$ and $m = n - 2$

$$\begin{aligned}
E\big(X(X-1)\big) &= \sum_{x=0}^{n} x(x-1)\tbinom{n}{x} p^x (1-p)^{n-x} \\
&= \sum_{x=0}^{n} x(x-1)\tfrac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \\
&= \sum_{x=2}^{n} \tfrac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x} \\
&= n(n-1)p^2 \sum_{x=2}^{n} \tfrac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x} \\
&= n(n-1)p^2 \sum_{y=0}^{m} \tfrac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \\
&= n(n-1)p^2 \big(p + (1-p)\big)^m \\
&= n(n-1)p^2
\end{aligned}$$

So the variance of X is

$E(X^2) - E(X)^2 = E(X(X - 1)) + E(X) - E(X)^2 = n(n - 1)p^2 + np - (np)^2$

$= np(1 - p)$

:: Q 6 ::

Answer

Bi-nominal frequency distribution:-

if the bi-nominal probability distribution is multiplied by N, the number of experiments or sets, the resulting distribution is known as the bi-nomial frequency distribution.

Formula:-

$$\boxed{N \binom{n}{x} p^x q^{n-x}}$$

Bi-nominal distribution:-

Many experiment consist of repeated independent trials each trial having two possible outcome. if the probability of each outcome remains the same throughout the trials then such trials are called "Bernouli trials" and

Date:

(Q7)

Answer

Coefficient of variation.

for Data Set A:

$$CV = \frac{3}{45} \times 100$$

$$\boxed{CV = 6.7}$$

for Data Set B:-

$$CV = \frac{11}{60} \times 100$$

$$\boxed{CV = 18.3}$$

for Data Set C:-

$$CV = 5/50 \times 100$$

$$\boxed{CV = 10}$$

for Data Set D:-

$$CV = 15/95 \times 100 \Rightarrow \boxed{60}$$