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I.D # 14596

SEMESTER : 4th BS (SE)

SECTION : B

SUBJECT : LINEAR ALGEBRA

INSTRUCTOR : SHAKEEL SIR

Q1:-

$$A = \begin{bmatrix} 1 & 2 & 2nd.\ ID \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Adjoint = ?

SOLUTION:-

2nd-ID = 4

So

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Now we find cofactors of each element.

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 1(6-1) = 5$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = -1(4-3) = -1$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = 1(2-9) = 7$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = -1(4-4) = 0$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix} = 1(2-12) = -10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -1(1-6) = 5$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} = +1(2-12) = -10$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix} = -1(1-8) = 7$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1(3-4) = -1$$

Now all cofactors change its each element.

$$A' = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A' = \begin{bmatrix} 5 & -1 & -7 \\ 0 & -10 & 5 \\ -10 & 7 & -1 \end{bmatrix}$$

$$A_{adj} = A'^{trans} = \begin{bmatrix} 5 & 0 & -10 \\ -1 & -10 & 7 \\ -7 & 5 & -1 \end{bmatrix} \text{ transpose}$$

$$A_{adj} = \begin{bmatrix} 5 & 0 & -10 \\ -1 & -10 & 7 \\ -7 & 5 & -1 \end{bmatrix} \text{ Ans}$$

Q1:-

ii) $B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$

$A_{adj} = ?$

SOLUTION:-

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix} = (1)(-8+16) = 8$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix} = (-1)(16-40) = 24$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = (1)(-4+5) = 1$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix} = (-1)(32+10) = -42$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix} = (1)(24-25) = -1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = (-1)(-6-20) = 26$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = (1)(32+5) = 37$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = (-1)(24-10) = -14$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = (+1)(-3-8) = -11$$

Now we replace all values in the cofactors.

$$B = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$B = \begin{bmatrix} 8 & 24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

Now transpose –

$$B_{adj} = B^{t} = \begin{bmatrix} 8 & -42 & 37 \\ 24 & -1 & -14 \\ 1 & 26 & -11 \end{bmatrix} \quad \underline{\text{Ans}}$$

Q2:- Find the cofactors of A_{21}, A_{31}, A_{33}

if $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & -3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$

SOLUTION:-

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = (-1)(-4+9) = -5$$

$$\boxed{A_{21} = -5}$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = (1)(-2-9) = -11$$

$$\boxed{A_{31} = -11}$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} = (1)(3-4) = -1$$

$$\boxed{A_{33} = -1}$$

Q 3:-

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \; \& \; I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

SOLUTION:-

$$\boxed{|A - dI| = 0} \rightarrow \text{formula}$$

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} - d \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} - \begin{vmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{vmatrix} = 0$$

$$\begin{vmatrix} 2-d & 1 & 1 \\ 1 & 3-d & 2 \\ -1 & 1 & 2-d \end{vmatrix} = 0$$

Now take Determinent

$$2-d\left((3-d)(2-d)-2\right) - 1\left(2-(2-d)\right)+1)' + (1+1(3-d) = 0$$

$$2-d\,(6-3d-2d+d^2-2) - 1\,(2-2+d) + 1\,(1+3-d = 0$$

$$2-d\,(d^2-5d+4) - 1\,(d) + 1\,(4-d) = 0$$

$2d^2 - 10d + 8 - d^3 + 5d^2 - 4d - d + 4 - d = 0$

By ordering

$-d^3 + 7d^2 - 16d + 12 = 0$

xing by $(-)$

$d^3 - 7d^2 + 16d - 12 = 0 \longrightarrow$ (A)

Now put $d - 2 = 0$

then $d = 2$ in eqn (A)

$d^3 - 7d^2 + 16d - 12 = 0$

put $d = 2$

$(2)^3 - 7(2)^2 + 16(2) - 12 = 0$

$8 - 7(4) + 32 + 12 = 0$

$8 - 28 + 32 + 12 = 0$

$-20 + 20 = 0$

$\boxed{0 = 0}$

So Eigon value $\therefore d = 2$ or $d - 2 = 0$

Ans