Q#1)
(a)
Ans) **Drag:-**

A body which is wholly or partially immersed in a homogenous fluid may be subjected to two kinds of forces arising from relative motion between body and fluid. These forces are termed as drag and lift. If the forces parallel to the motion then it is termed as drag force.

There are two components:-

1) Pressure Drag (F_P):-

It is equal to integration of components in direction of motion of all pressure forces exerted on surface of body.

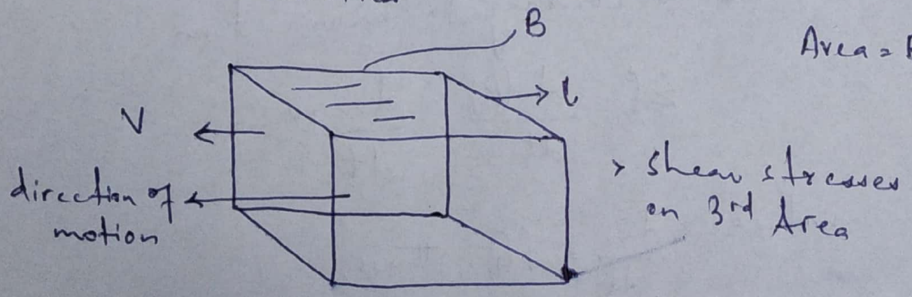
$$F_P = C_P \rho \frac{V^2}{2} A$$, where C_P depends on shape.

2) Friction Drag (F_f):-

It is equal to integration of components of shear stress along surface of body in direction of motion.

$$F_f = \underbrace{C_f \rho \frac{V^2}{2}}_{\text{shear stress}} BL$$

Area = BvL



*) Friction drag of boundary layer :-

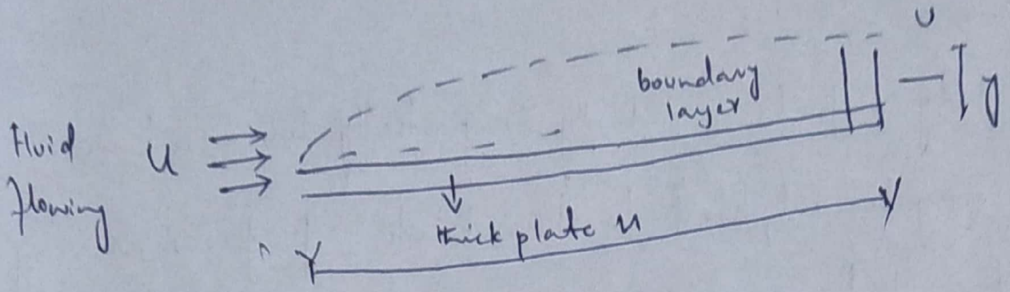
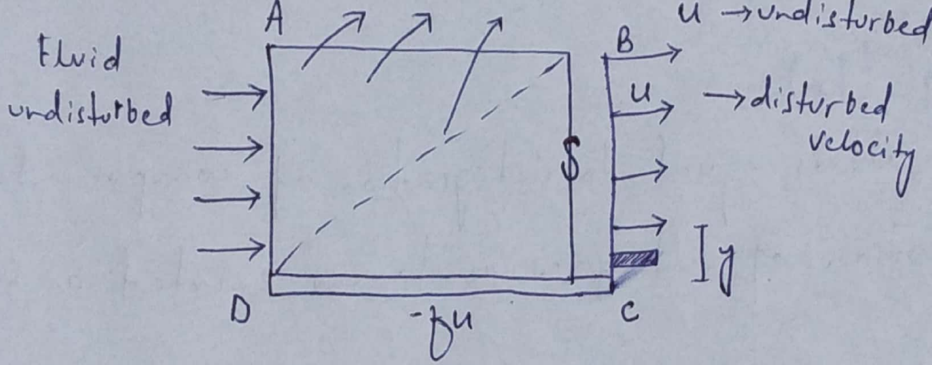


Fig. shows growth of boundary layer along one side of smooth plate inside the fluid.

Now consider a control volume



where δ is the thickness of boundary layer and U is undisturbed velocity. Thus $-F_n$ = drag = (rate in momentum in u-direction)

leaving through BC + rate of momentum through AB) - rate of momentum entering through DA)

$$\Delta P = P_{out} - P_{in}$$

Thus according to momentum

$$\Sigma f = \frac{d(P)}{dt} = \frac{dmv}{dt}$$

where $\frac{dm}{dt} = \rho Q$ thus,

$F = \rho Q V$

or

$F = \rho A . V . V$

$F = \rho A V^2$

$DA \rightarrow \rho U \, (U B \delta)$

$BC \rightarrow \rho B \int_0^\delta u^2 . dy$

$AB \rightarrow \rho U \, (U B \delta - B \int_0^\delta u . dy)$

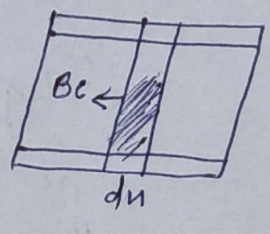
Putting value

$F_x = \rho B \int_0^\delta u \, (U - u) \, dy$

$F_x = \rho B U^2 \delta \alpha$ (where α is function of boundary layer)

Now to find local wall shear stress

$$\tau_0 = \frac{dF_x}{B . dx - \text{area}}$$

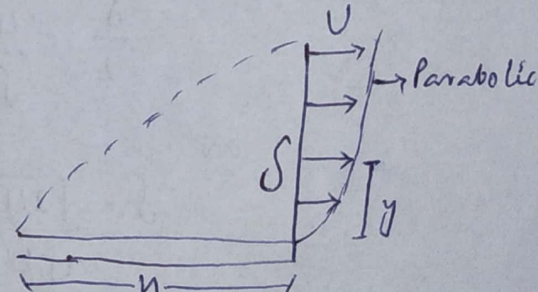


$F_x = \rho B U^2 \delta \alpha$

$\tau_0 = \rho U^2 \alpha \dfrac{d\delta}{dx}$ is general equation of shear stress

*) Laminar Boundary Layer :-

$$\frac{u}{U} = F\left(\frac{y}{\delta}\right)$$



Assume

$\eta = \dfrac{u}{U} = F(u)$ or $u = U f(\eta)$

In case of laminar flow

$$\tau_0 = \mu \left(\frac{du}{dy} \right)$$

$$= \frac{\mu}{\delta} \left(\frac{du}{d\eta} \right) \Rightarrow \frac{\mu U}{\delta} \left[\frac{df(\eta)}{d\eta} \right]$$

solving the equation

$$\tau_0 = \frac{\mu U B}{\delta} \quad \text{--- (i)}$$

As general equation is $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

Equating both equation

$$\frac{\mu U B}{\delta} = \rho v^2 \alpha \frac{d\delta}{dx}$$

or

$$\delta \, d\delta = \frac{\mu B}{\rho U \alpha} dx$$

Integrating the equation

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} + c$$

Now at $x = 0$, $\delta = 0$ Thus $c = 0$

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x$$

or

$$\delta = \sqrt{\frac{2 \mu B}{\rho U \alpha} x} \quad \text{or} \quad \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

÷ing and ×ing by "x"

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu \cancel{x}}{\rho U}} \cdot \frac{x}{\sqrt{\cancel{x}} \cdot \sqrt{x}}$$

where

$\alpha = 0.135$

$B = 1.63$

$R_x = \frac{\rho U x}{\mu}$

$$\delta = \frac{4.91}{\sqrt{R_x}} \cdot x \quad \text{or} \quad \frac{\delta}{x} = \frac{4.91}{\sqrt{R_x}}$$

Now

$$\tau_0 = \frac{\mu U B}{\delta}$$

Thus putting value

$$\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

where R_x is local Reynold number

Now

$$F_D = B \int_0^l \underbrace{\tau_0}_{\text{stress}} dx$$

Putting values

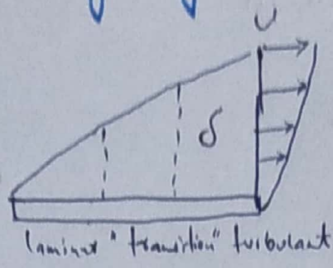
$$F_D = 0.664 B \sqrt{\rho \mu L U^3}$$

As general equation is

$$F_D = C_D \rho \frac{U^2}{2} BL \longrightarrow \text{equating both equation}$$

$$C_D = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \frac{1.328}{\sqrt{R}}$$

*) Turbulant Boundary layer:-



resistance is less so curve becomes straight.

Fig. show that velocity distribution in turbulant flow boundary layer shows a much steeper gradient near wall and flatter throughout remaining layer. The shear stress is greater in turbulant then in laminar layers.

As we have

$$\tau_0 = f\frac{\rho v^2}{8}$$

where v denotes average velocity of pipe

Now we have obtained an approximate relation between V and U by using pipe factor equation at

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{F}}$$

using friction factor of 0.028 from chart which is middle critical value.

So,

$$U = 1.235V$$

Now we have,

$$\tau_0 = f\rho\frac{v^2}{8}$$

As we know that

$$f = \frac{0.316}{R^{0.25}}$$

thus $$\tau_0 = \frac{0.316}{\left(\frac{Dv}{\nu}\right)^{1/4}} \cdot \frac{\rho v^2}{8}$$

where $v = \frac{U}{1.235}$ thus

$$\tau_0 = \frac{0.316}{\left(\frac{D}{\nu}\left(\frac{U}{1.235}\right)\right)^{1/4}} \cdot \frac{\rho}{8}\left(\frac{v}{1.235}\right)^2$$

$$\& \quad D = 2\delta$$

thus

$$\tau_0 = \frac{0.023\,\rho U^2}{\left(\frac{\delta U}{\nu}\right)^{1/4}}$$

As we have

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$

Equating both and integrating for boundary condition of $x = 0,\ \delta = 0$

thus

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{Ux}\right)^{1/5} x$$

for $\alpha = 0.0972$

$$\frac{\delta}{x} = \frac{0.377}{(R_x)^{1/5}}$$

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Putting values in equation

$$\tau_o = 0.0587\, \rho \frac{U^2}{2} \left(\frac{\nu}{Ux}\right)^{1/5}$$

Now

$$F_D = B \int_0^L \tau_o \, dx$$

$$F_D = 0.0735\, \rho \frac{U^2}{2} \left(\frac{\nu}{UL}\right) BL$$

As

$$F_D = C_D\, \rho \frac{U^2}{2} BL$$

equating both

$$C_D = \frac{0.0735}{R^{1/5}}$$

R is less than 10^7 for $500{,}000 \leq R < 10^7$

For $R > 10^7$

$$C_D = \frac{0.455}{(\log R)^{2.58}}$$

Answer.

Q1 (B)

As,

Specific energy, $E = y + \frac{v^2}{2g}$

The flow Q per unit width "b" can be expressed as

$$q = \frac{Q}{b}$$

Now average velocity will be

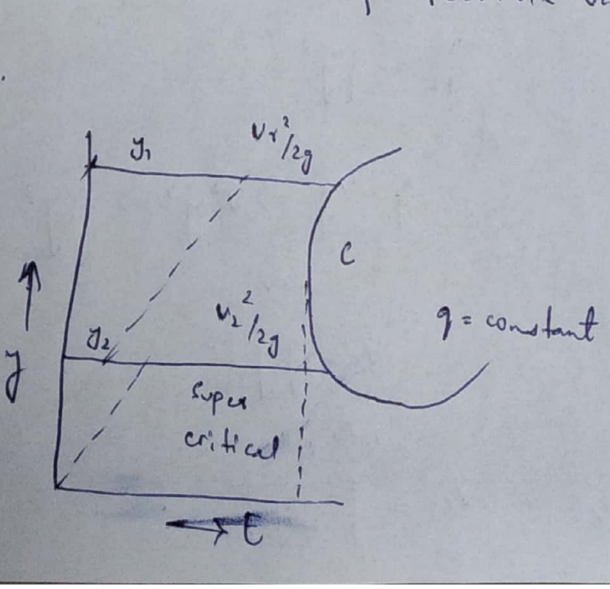
$$V = \frac{Q}{A} = \frac{qb}{by} = \frac{q}{y}$$

Thus

$$E = y + \frac{v^2}{2g} \Rightarrow y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

$$(E-y) = \frac{1}{2g}\left(\frac{q^2}{y^2}\right) \text{ or } (E-y)y^2 = \frac{q^2}{2g}$$

Thus plot of E vs y will be parabolic for particular q, there will be two kind of Possible values of y for a given E.



The equation is cubic with three roots being negative point c represents dividing point between two regime of flow thus for given q_1 and value of E is minimum, flow of that point is critical flow. Depth of flow at that point is critical. The y_c and velocity at that point is critical V_c. Thus,

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

for minimum S.E $\frac{dE}{dy} = 0$

Thus,

$$\frac{dE}{dy} = 1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right) = 0$$

$$\frac{q^2}{gy^3} = 1 \Rightarrow q^2 = gy^3$$

$$\frac{q^2}{g} = y^3 \Rightarrow \left(\frac{q^2}{g}\right)^{1/3} = y_{cr}$$

Now,

$$q_2 = gy^3$$

and

$$q = vy \Rightarrow v^2y^2 \Rightarrow gy^3$$

or

$$v^2 = gy_{cr}$$

or

$$\Rightarrow \quad V_{cr} = \sqrt{gy_{cr}}$$

Q2) GIVEN DATA :-

water flows = 3.5 m^3/s

bed slope = 0.0008.

n_1 = 0.0219

weidth of bed = 7.745 mm

*) Required :-

critical path = ?

critical velocity = ?

flow is critical sub flow or super crytical = ?

*) Solution :-

Manning equation

$$Q = \left(\frac{1}{n} R_n^{2/3} S_o^{1/2}\right)A \quad \text{---} (1)$$

Area = 7.745 × d

Parameter = d + 7.745 + d

Hydraulic radius $R_n = \frac{\text{Area}}{\text{Perimeter}}$

$$R_n = \frac{7.745(d)}{2d+7.745}$$

so we can put the values of "R_n" in eq (1)

$$Q = \left(\frac{1}{n} R_n^{2/3} S_o^{1/2}\right)A$$

Putting values

$$\Rightarrow 3.5 = \left(\frac{1}{0.0219} \times \left(\frac{7.745d}{2d+7.745}\right)^{2/3} \times (0.0008)^{1/2} \times 7.745d\right)$$

$$\Rightarrow \frac{3.5 \times 0.0219}{(0.0008)^{1/2}} = \left(\frac{7.745d}{2d+7.745}\right)^{2/3} \times 7.745d$$

$$\Rightarrow \left(\frac{3.5 \times 0.0219}{\sqrt{0.0008}}\right)^{3/2} = \frac{59.98d^2}{2d+7.745}$$

$$\Rightarrow 4.461(2d+7.745) = 59.98d^2$$

$$8.922d + 34.55 = 59.98d^2$$

$$59.98d^2 - 8.922d - 34.55 = 0$$

$$59.98d^2 - 8.922d = 34.55$$

$$51.058d = 34.55$$

$$d = \frac{34.55}{51.058}$$

$$\boxed{d = 0.676}$$

So The depth of channel is 0.676

Now,

As q = discharge per unit width

$$q = \frac{Q}{b}$$

$$q = \frac{3.5}{7.745}$$

$$\boxed{q = 0.451\ m^2/s}$$

⇒ critical depth y_{cr} :-

using equation

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_{cr} = \left(\frac{(0.451)^2}{9.81}\right)^{1/3}$$

$$\boxed{y_{cr} = 0.274\ m}$$

→ critical velocity, V_{cr} :-

using equation

$$V_{cr} = \sqrt{g y_{cr}}$$

$$V_{cr} = \sqrt{9.81 \times 0.274}$$

$$\boxed{V_{cr} = 1.63\ m/s}$$

$$V = \frac{Q}{A} = \frac{3.5}{7.745 \times 0.676}$$

$$\boxed{V = 0.668\ m}$$

$\Rightarrow \quad y = 0.676\,m, \quad y_{cr} = 0.274\,m, \quad V_{cr} = 1.63\,m/s$

$\Rightarrow$ As $y > y_{cr}$

and

$\Rightarrow V < V_{cr}$

So the flow is "Sub critical flow".

Q3) GIVEN DATA :-

width of smooth plate, $B = 200mm$
$= 0.2m$

length of smooth plate, $L = 800mm$
$= 0.8m$

Oil with specific gravity, $S = 0.89$
undisturbed velocity, $U = 5m/s$
kinematic viscosity, $\nu = 0.93 \times 10^{-4} m^2/s$

*) Required DATA :-

Friction drag on one side of a smooth plate
$F_D = ?$

*) Sol :-

check the flow

As $\Rightarrow$ $\nu = 0.93 \times 10^{-4} m^2/s$

$$R = \frac{LU}{\nu} = \frac{(0.85)(5)}{0.93 \times 10^{-4}}$$

$$R = 43010.75 < 500,000$$

Thus flow is laminar flow.

Now

$$\Rightarrow \quad C_f = \frac{1.328}{\sqrt{R}}$$

$$= \frac{1.328}{\sqrt{43010.75}}$$

$$C_f = 6.403 \times 10^{-3}$$

$$C_f = 0.0064$$

$$\Rightarrow \quad F_D = C_f \rho \frac{v^2}{2} BL$$

$$= (0.0064)(S_{oil} \times \gamma_{water}) \times \frac{(5)^2}{2} \times (0.2)(0.8)$$

$$= \left((0.0064)(0.89 \times 1000) \times \frac{5^2}{2} \times (0.2) \times (0.8) \right)$$

$$F_D = 11.392 \text{ N}$$

Ans.