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Semister	1st
Subject	linear Algebra
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Q1 Express the equation of plane passing through the point $A(2,-2,1)$, $B(-1,0,3)$, $C(5,-3,4)$

Sol:-> The Non parallel vector

$\vec{AB} = (-3, 2, 2)$

$\vec{AC} = (3, -1, 3)$

The perpendicular vector is

$n = \vec{AB} \times \vec{AC}$

$$n = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$n = i(6+2) - j(-9-6) + k(3-6)$

$n = 8i + 15j - 3K$

$n = (8, 15, -3)$

Now $p(x_0, y_0, z_0) = (2, -2, 1)$

$n(a, b, c) = (8, 15, -3)$

So Equation of plane is

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$$\Rightarrow \quad 8(x-2) + 15(x+2) - 3(z-1) = 0$$

$$\Rightarrow \quad 8x + 15y - 3z - 16 + 30 + 3 = 0$$

$$\Rightarrow \quad 8x + 15y - 3z + 17 = 0$$

Q1 (b) Part :-

Express a Pair of Plane whose intersection is given line,
$x = 2-3t$, $y = 3+t$, $z = 2-4t$

Sol:-> $x-2 = -3t$
÷ing both side by (-3)

$\Rightarrow \frac{x-2}{-3} = \frac{-\cancel{3}t}{-\cancel{3}}$

$\Rightarrow t = \frac{x-2}{-3}$

$y-3 = t$

$\Rightarrow t = y-3$

$z-2 = -4t$
÷ing b. side by (-4)

$\Rightarrow \frac{z-2}{-4} = \frac{-\cancel{4}t}{-\cancel{4}}$

$\Rightarrow t = \frac{z-2}{-4}$

So $\frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-2}{-4}$

for 1^{st} Plane take 1^{st} & 2^{nd}

$$\frac{x-2}{-3} = \frac{y-3}{1}$$

$\Rightarrow \quad x-2 = -3y+9$

$\Rightarrow \quad x+3y-11=0$

for 2nd plane tak 1st & 3rd

$$\frac{x-2}{-3} = \frac{z-2}{-4}$$

$= \quad -4x+18 = -3z+6$

or

$4x-3z-2=0$

Q2 $L(x,y) = (x+1, y, x+y)$
illustrate that L is the linear transformation.

$$L(x,y) = (x+1, y, x+y)$$

Let $u = (x_1, y_1)$ $v = (x_2, y_2)$

$$U+V = (x_2, x_2) + (x_2, y_2)$$

$$U+V = (x_1+x_2, y_1+y_2)$$

$$L(U+V) = L(x_1+x_2, y_2+y_2)$$

$$L(u+v) = (x_1+x_2+1, y_1+y_2, x_1+x_2+y_1+y_2) \quad -(1)$$

Given that $u = (x_1, y_1)$

$$L(u) = L(x_1, y_1) = (x_1+1, y_1, x_1+y_1)$$

$$L(v) = L(x_2, y_2) = (x_2+1, y_2, x_2+y_2)$$

$$L(u)+L(v) = (x_1+x_2+2, y_1+y_2, x_1+x_2+y_1+y_2)$$
$\hookrightarrow$ (ii)

Since $1 \neq 2$

So not L.T

Q3
using the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

then interpret to decode the message 77 54 38 71
49 29 68 51 33 76
48 40 86 53 52.

Solⁿ-> $A \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

77 54 38 71 49 29 68
51 33 76 48 40 86 53
52.

Inlant to Decodes.

$\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix}, \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix}, \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix}, \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix}, \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix}$

So Solve the Equation

$$L(x_1) = \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = Ax_1$$

for Since A is nonsingular

$$x_1 = A^{-1} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$$

Similarly

$$x_2 = A^{-1}\begin{bmatrix}71\\49\\29\end{bmatrix} = \begin{bmatrix}20\\15\\7\end{bmatrix}$$

$$x_3 = A^{-1}\begin{bmatrix}68\\51\\33\end{bmatrix} = \begin{bmatrix}18\\1\\16\end{bmatrix}$$

$$x_4 = \begin{bmatrix}76\\48\\40\end{bmatrix} = \begin{bmatrix}8\\16\\12\end{bmatrix}$$

$$x_5 = \begin{bmatrix}86\\53\\52\end{bmatrix} = \begin{bmatrix}1\\14\\19\end{bmatrix}$$

Using our corresponding between letter & Number we recieved the following massage.

PHOTOGRAPH PLANS

Answer

Q4 Find an equation of the plane passing through the point $(-1, 3, 2)$ & Perpendicular to vector $n = (0, 1, -3)$

Sol:-> $(-1, 3, 2)$ $n = (0, 1, -3)$

Equation of plane

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Given that

$$P = (x_0, y_0, z_0) = (-1, 3, 2)$$

$$n = (a, b, c) = (0, 1, -3)$$

So

$$\Rightarrow 0(x - (-1)) + 1(y - 3) - 3(z - 2) = 0$$

$$\Rightarrow 0(x + 1) + 1(y - 3) - 3(z - 2) = 0$$

$$\Rightarrow 0x + y - 3 - 3z + 6 = 0$$

$$\Rightarrow 0x + y - 3z - 3 + 6 = 0$$

$$\Rightarrow 0x + y - 3z + 3 = 0$$

$$\Rightarrow \boxed{y - 3z + 3 = 0}$$ -> Answer

Q5 Find an Eigen value & Eigen vector of Matrix

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

Since we know that

$$Ax = \lambda x$$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

~~then~~ ~~x~~

$$\Rightarrow \begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

then

$$x_1 + x_2 = \lambda x_1 \rightarrow (1)$$

$$= -2x_1 + 4x_2 = \lambda x_2 \rightarrow (2)$$

So $x_1 - \lambda x_1 + x_2 = 0$

$$\Rightarrow (1 - \lambda) x_1 + x_2 = 0$$

& $-2x_1 + 4x_2 - \lambda x_2 = 0$

$$= -2x_1 + (4 - \lambda) x_2 = 0$$

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$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Characteristic equation

$$\begin{vmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (1-\lambda)(4-\lambda)+2 = 0$

$\Rightarrow \quad 4-\lambda-4\lambda+\lambda^2+2=0$

$\Rightarrow \quad \lambda^2-5\lambda+6 = 0$

$\cancel{\Rightarrow \quad \lambda\, \lambda^2-5\lambda+6}$

$\Rightarrow \quad \lambda^2-3\lambda-2\lambda+6 = 0$

$\Rightarrow \quad \lambda(\lambda-3)-2(\lambda-3)=0$

$\Rightarrow \quad (\lambda-3)(\lambda-2)=0$

$\Rightarrow \quad \lambda-3=0 \qquad \lambda-2=0$

$\Rightarrow \quad \lambda_1 = 3, \quad \lambda_2=2$

Are Eigen values

Now find Eigen vectors of $\lambda_1 = 3$ put (i) & (2)

then $\quad x_1+x_2 = 3x_1 \quad \rightarrow (1)$

$= \quad -2x_1 + x_2 = 0$

$\Rightarrow \quad 2x_1 - x_2 = 0$

$\Rightarrow \quad -2x_1 + 4x_2 = 3x_2 \quad — \text{(ii)}$

$\Rightarrow \quad -2x_1 + x_2 = 0$

$\Rightarrow \quad 2x_1 - x_2 = 0$

$x_1 = \frac{1}{2} x_2$

let $x_2 = \gamma$

where $\gamma = 0$

So

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\gamma \\ \gamma \end{bmatrix}$$

Eigen vector for $\lambda_2 = 2$ put in (i) & (ii)

$x_1 + x_2 = 2x_1 \quad — \text{(i)}$

$-2x_1 + 4x_2 = 2x_2 \quad \rightarrow \text{(ii)}$

$= \quad -x_1 + x_2 = 0 \quad -\text{(i)}$

$\Rightarrow \quad x_1 - x_2 = 0$

$\Rightarrow \quad x_1 = x_2$

$= \quad -2x_1 + 4x_2 = 2x_2 \quad - \text{(ii)}$

$= \quad -2x_1 + 2x_2 = 0$

$= \quad x_1 - x_2 = 0$

$= \quad x_1 = x_2$

$x_1 = \gamma$ then $x_2 = \gamma$

So $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \gamma \\ \gamma \end{bmatrix}$