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Department	**BE CIVIL**
Subject	**Structure Analysis 2**
Submitted To	**Engr . Adeed Khan**

SUMMER FINAL EXAM .

QUESTION NO = 01

—{ QNo. 1 }—

Analyze the beam shown in Figure 1 by stiffness method. Assume EI is constant.

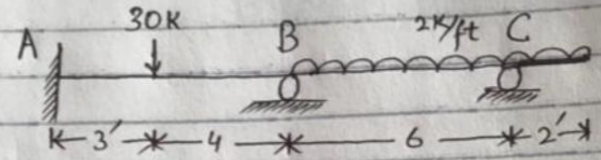
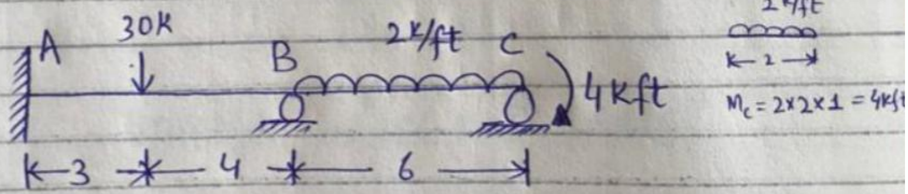
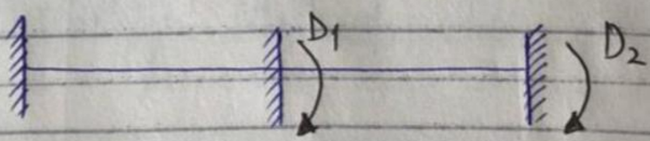


Figure 1

Step 1 : K.I = 2° (Neglecting Axial Effects)

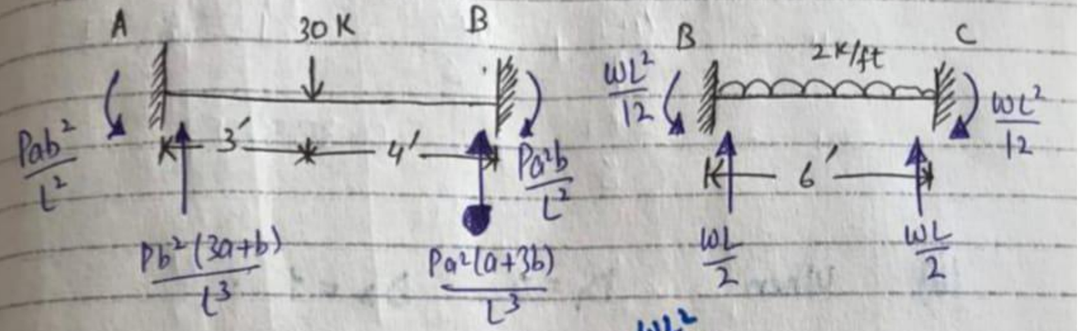
Step 2 : Select the unknown joint displacement.

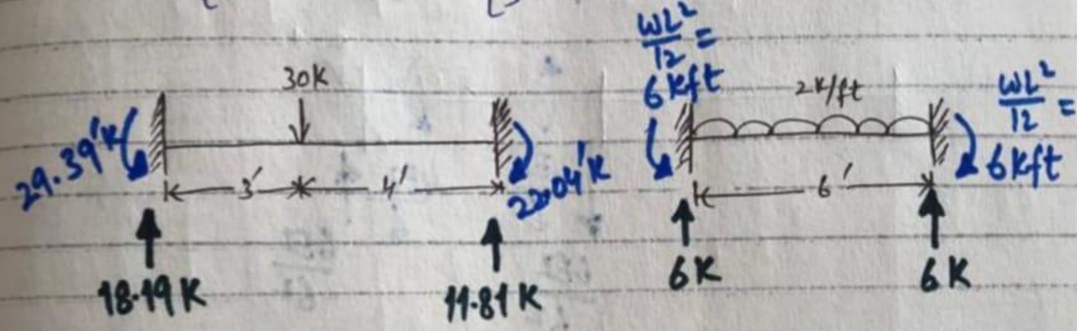




$$[D] = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad [AD] = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step 3 : Compute [ADL] matrix.





$ADL_1 = 22.04'K - 6'K$

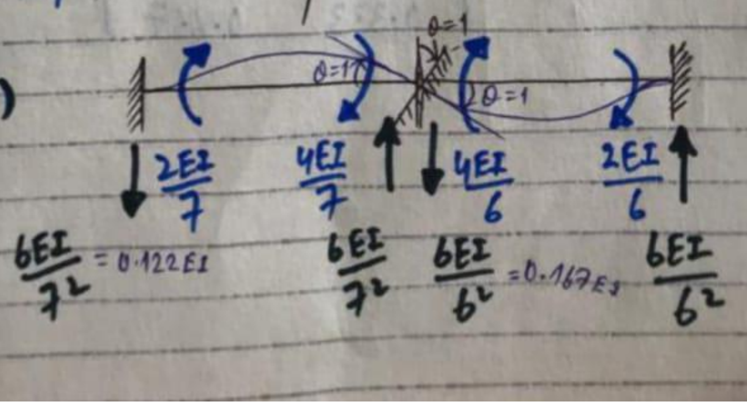
$ADL_1 = 16.04'K$

$ADL_2 = 6'K$

So, $[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} 16.04 \\ 6 \end{bmatrix}$

Step 4: Compute [S] matrix.

(i)

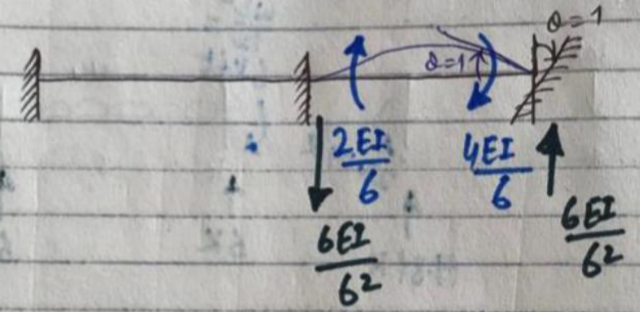


When $D_1 = 1$, $D_2 = 0$.

$$S_{11} = \frac{4EI}{7} + \frac{4EI}{6} = 1.238\ EI$$

$$S_{21} = \frac{2EI}{6} = 0.333\ EI$$

(ii) When $D_1 = 0$, $D_2 = 1$.



$$S_{12} = \frac{2EI}{6} = 0.333\ EI$$

$$S_{22} = \frac{4EI}{6} = 0.667\ EI$$

Stiffness Matrix $[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

$$[S] = \begin{bmatrix} 1.238 & 0.333 \\ 0.333 & 0.667 \end{bmatrix} EI$$

Step 5: Compute the values of D_1 & D_2.

$$[AD] = [ADL] + [S][D]$$

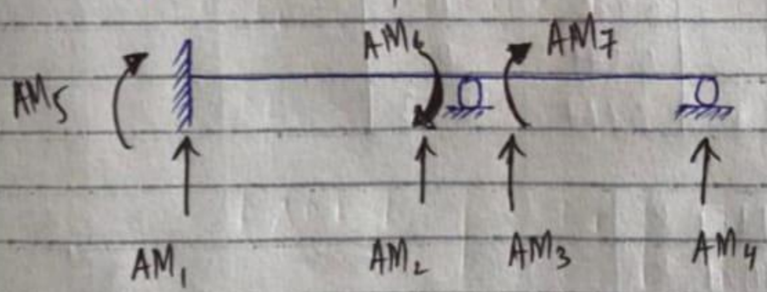
$$[D] = [S]^{-1} [AD - ADL]$$

$$[D] = \left(\begin{bmatrix} 1.238 & 0.333 \\ 0.333 & 0.667 \end{bmatrix} EI \right)^{-1} \begin{bmatrix} 0 - (16.04) \\ 4 - (6) \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} 0.933 & -0.466 \\ -0.466 & 1.732 \end{bmatrix} \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$[D] = \frac{1}{EI} \begin{bmatrix} -14.03 \\ 4.01 \end{bmatrix}$$

Step 6: Compute Member End Actions.



$$[AM] = [AML] + [AMD][D]$$

$[AML]$; $$[AML] = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} = \begin{bmatrix} 18.19\ K \\ 11.81\ K \\ 6\ K \\ 6\ K \\ -29.39'K \\ 22.04'K \\ -6'K \end{bmatrix}$$

12:06 pm

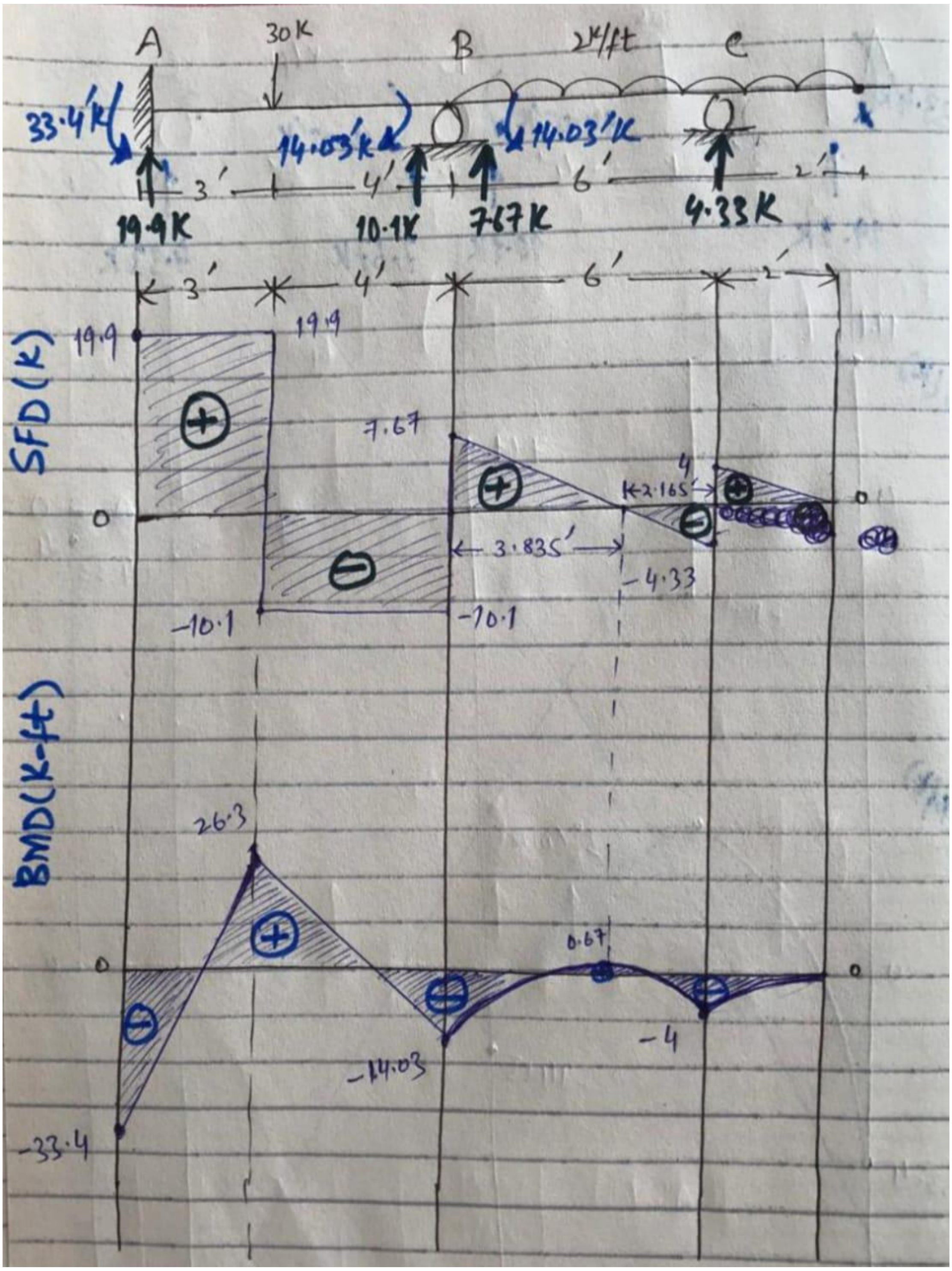
[AMD] ;

$$[AMD] = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{62} \\ AMD_{71} & AMD_{72} \end{bmatrix} = EI \begin{bmatrix} -0.122 & 0 \\ 0.122 & 0 \\ -0.167 & -0.167 \\ 0.167 & 0.167 \\ 0.286 & 0 \\ 0.571 & 0 \\ 0.667 & 0.333 \end{bmatrix}$$

Now; $[AM] = [AML] + [AMD][D]$

$$[AM] = \begin{bmatrix} 18.19 \\ 11.81 \\ 6 \\ 6 \\ -29.39 \\ 32.04 \\ -6 \end{bmatrix} + \begin{bmatrix} -0.122 & 0 \\ 0.122 & 0 \\ -0.167 & -0.167 \\ 0.167 & 0.167 \\ 0.286 & 0 \\ 0.571 & 0 \\ 0.667 & 0.333 \end{bmatrix} \cancel{EI} \times \frac{1}{\cancel{EI}} \begin{bmatrix} -14.03 \\ 4.01 \end{bmatrix}$$

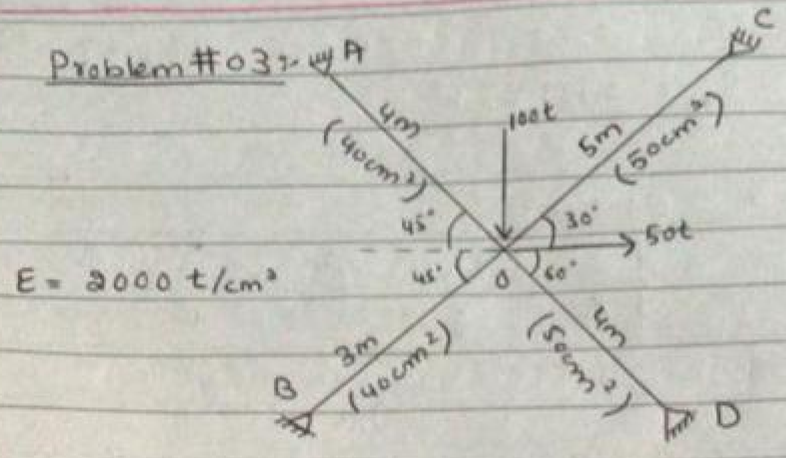
$$[AM] = \begin{bmatrix} 19.9\ k \\ 10.1\ k \\ 7.67\ k \\ 4.33\ k \\ -33.4\ 'k \\ 14.03\ 'k \\ -14.03\ 'k \end{bmatrix}$$



A
30K
B
2K/ft
C
33.4'K
14.03'K
14.03'K
3'
4'
6'
2'
19.9K
10.1K
7.67K
4.33K
SFD (K)
19.9
7.67
4
2.165'
3.835'
-4.33
-10.1
0
BMD (K-ft)
26.3
0.67
-14.03
-4
-33.4
0

QUESTION NO = 02

Problem#03:-



Sol: For A:

$$\sin 45^\circ = \frac{P}{h} = \frac{P}{4}$$

$\Rightarrow$ $P = 2.828$ m

$$\cos 45^\circ = \frac{b}{4}$$

$\Rightarrow$ $b = 2.828$ m

For B :-

$$\sin 45^\circ = \frac{P}{3}$$

$\Rightarrow$ $P = 2.12$ m

$$\cos 45^\circ = \frac{b}{h}$$

$\Rightarrow$ $b = 2.12$ m

For C :-

$$\sin 30^\circ = \frac{P}{h=5}$$

$\Rightarrow$ $P \Rightarrow 2.5$ m

$$\cos 30^\circ = \frac{b}{5}$$

$$\Rightarrow \quad b = 4.33\,m$$

Now $EA_{(A)} = 2000 \times 40 = 80{,}000\,t$

$EA_{(B)} = 2000 \times 40 = 80{,}000\,t$

$EA_{(C)} = 2000 \times 50 = 100{,}000\,t$

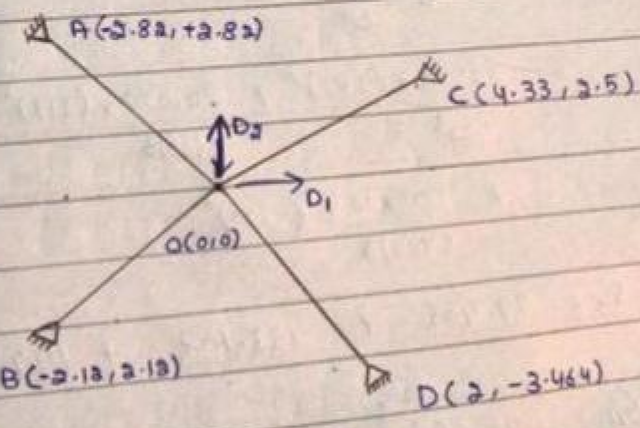
$EA_{(D)} = 2000 \times 50 = 100{,}000\,t$

Step#01:- K.I

$$K.I = 2j - r$$

$$= 2(5) - 8 = 2^\circ$$

Step#02: Select unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} AD_1 \\ AD_1 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step#03: $[AMD]_{4\times 2}$ & $[S]_{2\times 2}$

i) $D_1 = 1$, $D_2 = 0$

$$AMD = \frac{EA}{L^2}(X_k - X_j)$$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0+282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0+212) = 188.44$$

$$AMD_{31} = \frac{100,000}{(500)^2} \times (0-433) = -173.2$$

$$AMD_{41} = \frac{100,000}{(400)^2} \times (0-200) = -125$$

Now $S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3}(X_k - X_j)^2$

$$= \frac{80,000 \times (282)^2}{400^3} + \frac{80,000 \times (212)^2}{(300)^3} + \frac{100,000 \times (-433)^2}{(500)^3}$$

$$+ \frac{100,000 \times (-200)^2}{(400)^3}$$

$S_{11} = 99.405 + 133.167 + 149.991 + 62.5$

$$\boxed{S_{11} = 445.063}$$

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (X_k - X_j)(Y_k - Y_j)$$

$$= \frac{80{,}000}{(400)^3} \times (282)(-282) + \frac{80{,}000}{(300)^3} \times (212)(212)$$

$$+ \frac{100{,}000}{(500)^3} \times (-433)(0-250) + \frac{100{,}000}{(400)^3} \times (-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

ii) $D_1 = 0$, $D_1 = 1K'$

$$AMD = \frac{EA}{L^2}(Y_k - Y_j)$$

$$AMD_{12} = \frac{80{,}000}{400^2}(-282) = -141$$

$$AMD_{22} = \frac{80{,}000}{300^2}(212) = 188.44$$

$$AMD_{32} = \frac{100{,}000}{500^2}(-250) = -100$$

$$AMD_{42} = \frac{100{,}000}{400^2}(346) = 216.25$$

Now, $S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}(Y_k - Y_j)^2$

$$= \frac{80{,}000}{400^3}(-282)^2 + \frac{80{,}000}{300^3}(212)^2 + \frac{100{,}000}{500^3}(-250)^2$$

$$+ \frac{100{,}000}{400^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step #04:

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

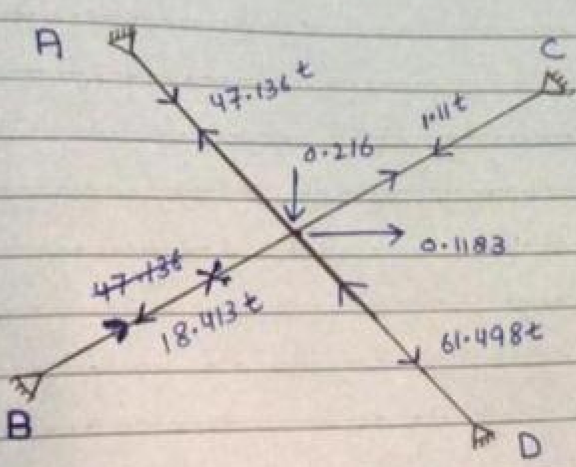
Step#06:- $[AM]$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

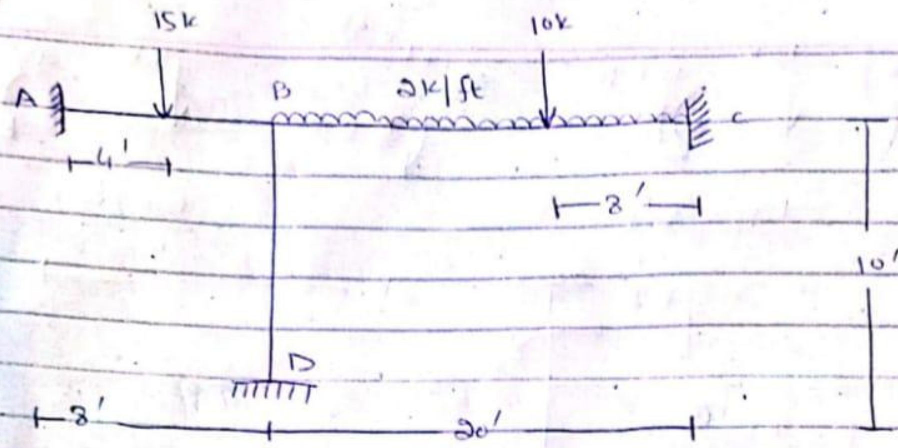
$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$



QUESTION NO = 03



Sol

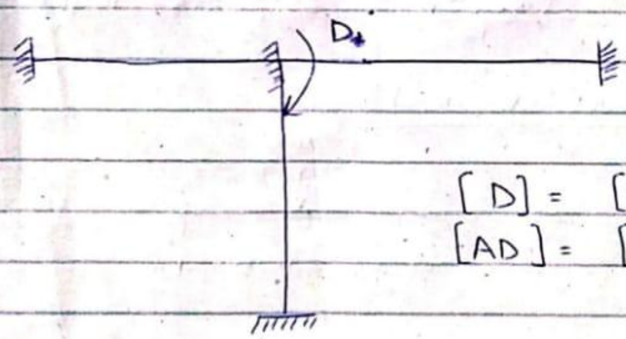
Step #1 :-

Determine Kinematic Indeterminancy .

K·I = 1°

Step #2 :-

Determine Unknown Joint Displacement.

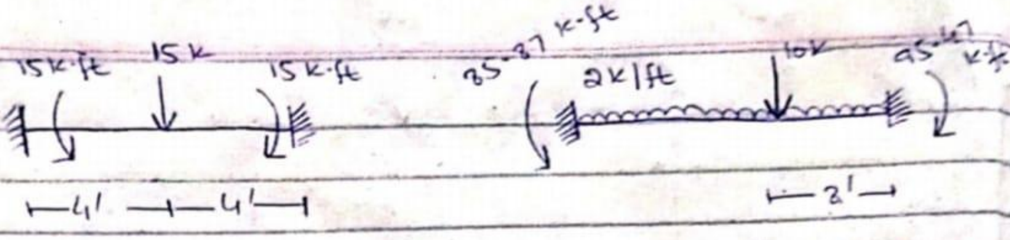


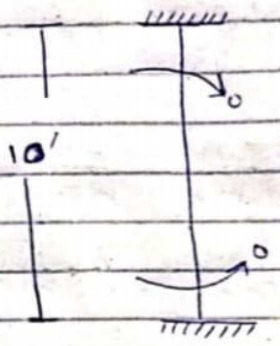
[D] = [?]

[AD] = [0]

Step #3 :-

Compute [ADL] Matrix .





=> Point Load at Center :-

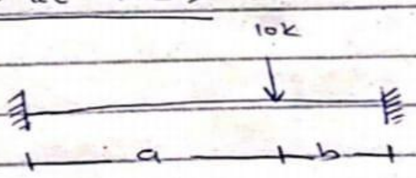
$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8} = 15 \text{ kip.ft}$$

=> Uniformly Distributed Load :-

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ k.ft}$$

=> Point Load (Not at mid) :-

Suppose :-



For Left End :-

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ k.ft}$$

For ~~Left~~ Right End :-

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ k.ft}$$

So Total Moment at left end :-

$19.2 + 66.67 = 85.87$ k.ft

Similarly at right End :-

$28.8 + 66.67 = 95.47$ k.ft

So $[AD_L] = -85.87 + 15 = -70.87$ k.ft

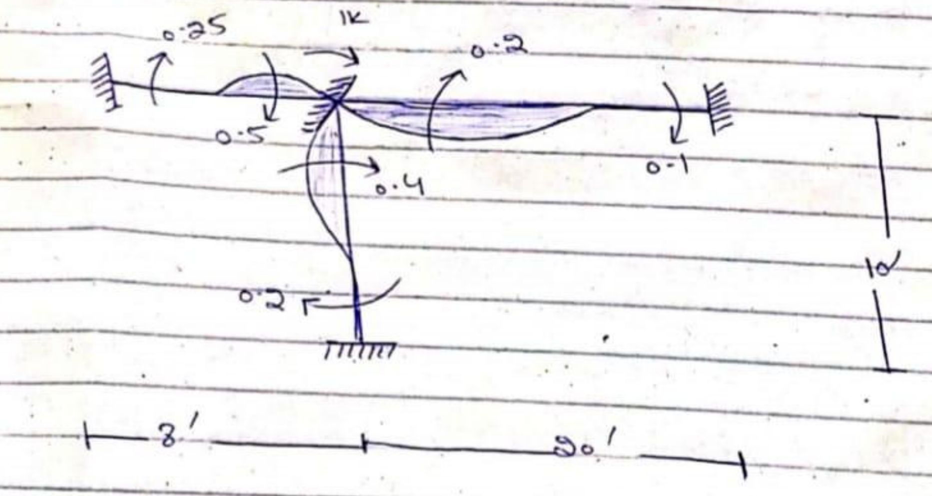
Step #4 :-

Determine [S] Matrix

$[S] = [S_{11}]$

Now,-

D = 1k



$\Rightarrow \frac{4EI}{8} = 0.5 \qquad \frac{2EI}{8} = 0.25$

$\Rightarrow \frac{4EI}{20} = 0.2 \qquad \frac{2EI}{20} = 0.1$

$\Rightarrow \frac{4EI}{10} = 0.4 \qquad \frac{2EI}{10} = 0.2$

$[S] = (0.5 + 0.4 + 0.2)\ EI$

$= 1.1\ EI$

$[S] = 1.1\ EI$

<u>Step #5 :-</u>

Compute [D] Matrix

$[D] = [S]^{-1} \times [AD] - [ADL]$

$[D] = \frac{1}{1.1} \times [0] - [-70.87]$

$= \frac{70.87}{1.1}$

$[D] = [64.42]\ ^{1}/_{EI}$

THE END