Q#1 :-

(i) The order of Matrix AB is $\underline{m \times n}$.

(ii) The number of non-zero rows in on Echelon form is Rank of matrix.

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = \underline{8}$.

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= -2i^2 - i^2$$

$\because i^2 = -1$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$= \underline{3}. \quad \text{Ans.}$$

(v) The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is scalar matrix.

The give matrix A is a scalar matrix because the diagonal elements are same and non-diagonal are zero.

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(vii) The order and degree of differential equation. $\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is

Sol:-

order = $\underline{1}$.

Degree = $\underline{3}$.

(viii) The order and degree of differential equation.

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is}$$

Sol:-

Order = $\underline{\text{Two}}$.

Degree = $\underline{\text{one}}$.

(x)

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expand by C_1

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$\Rightarrow 1(bc^2 - cb^2) - 1(ac^2 + ca^2) + 1(ab^2 - a^2b)$

$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b$

$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$

$= a^2c - a^2b + ab^2 - cb^2 - ac^2 - bc^2$

$= \boxed{a^2(c-b) + b^2(a-c) + c^2(b-a)}$ **Ans.**

(ix) The differential equation $2\frac{dy}{dx} + x^2 y = 2x+3$

$y = (0) = 5$ is ______.

Sol:-

$$2\frac{dy}{dx} + x^2 y = 2x+3$$

$$2y' + x^2 y = x^2+3 \quad , \quad y(0) = 5$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{x^2+3}{2}$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{1}{2}(x^2+3)$$

$$\mu = \frac{x^2}{2}$$

$$e^{\int \frac{x^2}{2} dx} = e^{x^3/6}$$

$$e^{x^3/6} y' + e^{x^3/6}\left(\frac{x^2}{2}\right)y = \frac{1}{2} e^{x^3/6}(x^2+3)$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6} + C}{2e^{x^3/6}}$$

$$y(0) = \frac{0+3}{2} = \frac{3}{2}$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + \frac{3}{2}$$

Ans

(vi) Solution of $\frac{dy}{dx} + 2xy = y$?

Sol:-

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x) \qquad \left(\begin{array}{l}\text{Taking} \\ y \text{ common}\end{array}\right)$$

$$\frac{dy}{dx} = y(1-2x)$$

$$\frac{dy}{y} = (1-2x)\,dx$$

take integration

$$\int \frac{1}{y}\,dy = \int (1-2x)\,dx$$

$$\ln y = \int 1\,dx - \int 2x\,dx$$

$$\Rightarrow \ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\ln y = x - x^2 + c$$

$$\ln y = x - x^2 + c$$

$$e^{\ln y} = e^{x-x^2+c}$$

$$\boxed{y = e^{x(1-x)+c}}$$

Ans.

Q # 2. (A)

Express the Determinant:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the Product of factor's which are linear in a, b, c -

Sol :-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

common $(a\ b\ c)$

$$\Rightarrow abc(bc^2 - b^2c - ac^2 + ab^2 - a^2b)$$

Ans : $\Rightarrow \boxed{abc[bc(c-b) - ac(c+a) + ab(b-a)]}$

Q # 2. B

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic equ $\rightarrow |A - \lambda I| = 0 \rightarrow$ (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take derterminent

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$- 1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow \text{(B)}$$

Again $\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$ Expand by R_1

$$\Rightarrow \quad 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[\big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + \big((-1)(2-\lambda) - (-1)(-1)\big)$$
$$- 1\big((-1)(-1) - (-1)(3-\lambda)\big)\Big]$$

$$= (3-\lambda)(6 - 3\lambda - 2\lambda + \lambda^2 - 1) + (-2 + \lambda - 1) - (+1 + 3 - \lambda)$$

$$= (3-\lambda)(\lambda^2 - 5\lambda + 5) + (-3 + \lambda) - (4 - \lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \rightarrow (a)$$

$$\Rightarrow \quad +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow \quad -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow \quad -1(6 - 3\lambda - 2\lambda + \lambda^2 - 1) + 1(-2 + \lambda - 1)$$

P.T.O

$$\Rightarrow \quad -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$= \quad \boxed{-\lambda^2 + 6\lambda - 8} \longrightarrow (b)$$

$$\Rightarrow \quad -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1.

$$-\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0 \right]$$

$$\Rightarrow \quad -\left[-(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1) \right]$$

$$\Rightarrow \quad -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= \quad -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$= \quad \boxed{-\lambda^2 + 6\lambda - 8} \longrightarrow (c)$$

Put eq (a), (b) and (c) in (B)

$$= \quad (2-\lambda)\left[-\lambda + 8\lambda^2 - 18\lambda + 8 \right] - \lambda^2 + 6\lambda - 8 - \lambda^2 + 6\lambda - 8.$$

$$= \quad -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda - \lambda^2 + 6\lambda - 8$$
$$-\lambda^2 + 16\lambda - 8$$

$$= \quad \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda - 8\lambda + 6\lambda$$
$$+ 6\lambda + 16 - 16$$

$$\Rightarrow \quad \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$$

By synthatic division

we get :-

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0 \Rightarrow \boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0$$

By factorizatiom method.

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$(\lambda-4)(\lambda-4)=0$$

$$\lambda=4\ ,\ \lambda=4$$

$$\boxed{\lambda_1=0\ ,\ \lambda_2=2\ ,\ \lambda_3=4\ ,\ \lambda_4=4}$$

Ans.

Q # 3. $(x^2+3y^2)\,dx - 2xy\,dy = 0$

$x = 2, \quad y = 6$

Sol:-

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$\Rightarrow \quad (x^2+3y^2)\,dx = 2xy\,dy$$

Dividing both side by $2xy\,dy$

we get :-

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{\cancel{x^2}}{2\cancel{x}y} + \frac{3\cancel{y^2}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \longrightarrow (*)$$

Let $y = vx$

Diff :- $dy = v\,dx + x\,dv$

Dividing by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \longrightarrow (a)$$

Put (a) in (*)

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\,\frac{v\cancel{x}}{\cancel{x}}\right]$$

P.T.O

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying Both sides by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying both sides by $\frac{dx}{dv}$

we get :-

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multiplying both sides by $\frac{v}{x(1+v^2)}$

we get :-

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on both sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

Put $v = y/x$

$$1 + (y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3 c \longrightarrow (11)$$

Put $x=2$, $y=6$ in eq (11)

$$4 + 36 = 8c$$

$$c = \frac{\cancel{40}\,5}{\cancel{8}}$$

$\boxed{c = 5} \rightarrow$ Put in (11)

So

$$x^2 + y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Taking "$\sqrt{\ }$" on both sides

$\boxed{y = +x\sqrt{5x-1}}$, $\boxed{y = -x\sqrt{5x-1}}$

or

Ans:- $\boxed{y = \pm x\sqrt{5x-1}}$

compt..