Subject:-

Differential Equations

Submitted BY:-

Umair Niaz

ID# 7910

Submitted To:-

Shomaila Mazhar.

Q NO . 1

(1)

Solve the following objective type questions.

1) The order of matrix A is m x p and the order of B is p x n then the order of matrix AB is?

Sol:- The order of matrix is equal to the no of its row multiply by no of column

So,

$A = m \times p$ has "m" no of rows and "p" no of column.

Similarly, $B = p \times n$

then its "p" no of Rows and n has no of column

Also the number of column in A is equal to the no of rows in B so there matrix are comfortable for multiplication and there order will be

$$\boxed{AB = m \times n}$$

(ii) The number of non-zero rows in Echelon form? (2)

Sol:-

$\boxed{\text{ONE}} \Rightarrow$ Ans.

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Sol:- for Singular Matrix $|R| = 0$

So, $|B| = 1 \times a - 4 \times 2 = 0$

$= a - 8 = 0$

So value of $a = 8$

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol:-

$A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$= (2i)(-i) - (i)(i)$

$= -2i^2 - i^2$

we know that $i^2 = -1$

$= -2(-1) - (-1)$

$= 2 + 1$

$\boxed{|A| = 3}$

Scanned with CamScanner

(v) The matrix $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ is ?

Sol:- If each element of a principal diagonal of a matrix is some non-zero scalar and all other elements are zero then it is a scalar matrix so,

$A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ is a scaler Matrix

(vi) Solution of $\frac{dy}{dx} + 2xy = y$?

Sol:.

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$

$$\frac{1\,dy}{y} = (1-2x)\,dx$$

$$\int \frac{1\,dy}{y} = \int (1-2x)\,dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\ln y = x - x^2 + c$$

(vii) The order and degree of differential equation.

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ is?}$$

Sol:-

The order of differential equation is the order of highest derivatives known as differential co-efficient and Degree is the power of highest derivative so,

order = 1

degree = 3

(viii) The order and degree of differential equation.

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx}\right) \text{ is?}$$

Sol:-

order = Two

Degree = one

(ix) The differential equation $2\frac{dy}{dx} + x^2y = 2x+3$, $y(0) = 5$ is? (5)

Solution:

$$2\frac{dy}{dx} + x^2y = 2x+3$$

$$2dy + x^2y = (2x+3)\,dx$$

$$2dy = (2x+3-x^2y)\,dx$$

$$\int 2dy = \int (2x+3-x^2y)\,dx$$

$$2y = \frac{\cancel{2}x^2}{\cancel{2}} + 3x - y\frac{x^3}{3} + C$$

$$2y = x^2 + 3x - \frac{x^3y}{3} + C$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + C \quad \text{(i)}$$

Put $x=0$, $y=5$

$$5 = 0+0-0+C$$

$$5 = C$$

or

$$C = 5$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + 5$$ Ans

(x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ (6) is ?

Sol:-

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expand by R_1

$$|A| = +1\begin{vmatrix} b & b \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= 1(bc^2 - b^2c) - a(c^2 - b^2) + a^2(c - b)$$

$$|A| = bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$$

Q No — 2 (A-Part)

Express the Determinent ①

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the Product of factors which are linear in a, b, c

Sol:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^2 + a^3cb^3 - a^3b^2c$$

common abc

$$\Rightarrow abc(bc^2 - b^2c - ac^2 - a^2c + ab^2 - a^2b)$$

$$\Rightarrow abc\{bc(c-b) - ac(c-a) + ab(b-a)\}$$

Ans.

Scanned with CamScanner

Q No - 2 (B- Part)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic equ $\rightarrow |A - \lambda I| = 0$ — A

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take deteminent

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} \quad 2$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \quad — \quad B$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 2-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} \text{ expand by } R_1$$

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda)\Big[\big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + 1\big((-1)(2-\lambda)$$

$$-(-1)(-1)\big) - 1\big((-1)(-1) - (-1)(3-\lambda)\big)\Big]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda^2 - 18 + 8} \quad — 9$$

Scanned with CamScanner

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} \quad 3$$

Expand by C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \rightarrow b$$

$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 3-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)$$

$$\Rightarrow -(3-\lambda+\lambda^2-5\lambda+5)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \longrightarrow c$$

Put a, b and c in B

$$(2-\lambda)[-\lambda^3+8\lambda^2-18\lambda+8]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$\Rightarrow -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^3 - 8\lambda^3 + 18\lambda^2 - 8\lambda$
$- \lambda^2 + 6\lambda - 8 - \lambda^2 + 16\lambda - 8$

$\Rightarrow \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda - 8\lambda + 6\lambda$
$+ 6\lambda + 16 - 16$

$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$ (4)

By synthetic division we get:-

$\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$

$(\lambda = 0)$

$\lambda - 2 = 0 \Rightarrow \boxed{\lambda = 2}$

$\lambda^2 - 8\lambda + 16 = 0$

By factorization Method:-

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$

$(\lambda - 4)(\lambda - 4) = 0$

$\lambda = 4, \lambda = 4$

$$\boxed{\lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 4, \lambda_4 = 4}$$

Ans

QNO—3 (1)

$(x^2+3y^2)\,dx - 2xy\,dy = 0$

$x=2 \quad , \quad y=6$

Sol:-

$(x^2+3y^2)\,dx - 2xy\,dy = 0$

$(x^2+3y^2)\,dx = 2xy\,dy$

Divide both sides by $2xy\,dx$ ~~[illegible]~~

we get;

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \quad \text{—— (1)}$$

Diff:

$dy = v\,dx + x\,dv$

Dividing by dx

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \quad \text{—— (a)}$$

Put a in (1)

$$v + \frac{x\,dv}{2x} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right)$$

(2)

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying both sides by 2

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v} dx$$

Multiplying both sides by $\frac{dx}{dv}$ we get

$$2x\,dv = \frac{1+v^2}{v} dx$$

Multiplying both sides by $\frac{v}{x(1+v^2)}$

we get

$$\frac{v}{(1+v^2)} dv = \frac{1}{x} dx$$

Take $\int$ on both sides.

$$\int \frac{2v}{1+v^2} dv = \int \frac{1}{x} dx + C$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

$= \quad 1+v^2 = xc$

put $v = y/x$

$1+(y/x)^2 = xc$

$\frac{x^2+y^2}{x^2} = xc$

$x^2+y^2 = x^3c$ — (ii)

put $x=2$, $y=6$ in eq (ii)

$4 + 36 = 8c$

$c = \frac{40}{8} = 5$ — put in eq (ii)

So,

$x^2+y^2 = 5x^2$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x-1)$

Taking "$\sqrt{\ }$" on both sides

$y = +x\sqrt{5x-1}$, $y = -x\sqrt{5x-1}$

$\boxed{y = \pm x\sqrt{5x-1}}$ Ans.