12.509

Q2 Page 01

Q1

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$

Sol:

$$\lim_{(x,0)} = \frac{x(0)}{\sqrt{x^2+0^2}}$$

$$\lim_{(x,0)} = \frac{x}{\sqrt{x^2}} = \frac{x}{x} = 1$$

Now

$$\lim_{(0,y)} = \frac{0\,y}{\sqrt{0^2+y^2}}$$

$$\lim_{(0,y)} = \frac{y}{\sqrt{y^2}} = \frac{y}{y} = 1$$

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Page #02

Q2

Q2

$$f(x,y) = \begin{cases} \frac{\sin(x^2+y^2)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ \text{if } (x,y) = (0,0) \end{cases}$$

Solution:

Find $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$

$x = r\cos\theta$

$y = r\sin\theta$

$r^2 = x^2+y^2$

as $(x,y) \to (0,0)$

$r \to 0$

$\lim_{r\to 0} \frac{\sin(r^2)}{r^2}$

$\sin(0) = 0$

L'Hôpital $= \lim_{r\to 0} \frac{\cos(r^2)\cdot 2r}{2r}$

$2 \lim_{r\to 0} \cos(r^2) = \cos(0)$

$= 1$

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Q3 Page # 03

Q2

Function $f(x,y) = e^x \sin y + e^y \cos x$

Sol:

$$f(x,y) = e^x \sin y + e^y \cos x$$

$$\frac{\partial f(x,y)}{\partial y} = e^x \sin y + e^y \cos x$$

$$\frac{\partial f}{\partial y} = e^x \{x \sin y + e^y \cos x\}$$

$$\frac{\partial f(x,y)}{\partial y} = \sin y e^x (1) + e^y (-\sin x)$$

$$\frac{\partial f(x,y)}{\partial x} = e^x \sin y - e^y \sin x$$

$$\frac{\partial^2 f(x,y)}{\partial x^2} = e^x (1) \sin y - e^y \cos x (1)$$

$$\frac{\partial^2 f(x,y)}{\partial x^2} = e^x \sin y - e^y \cos x \quad \text{---(1)}$$

$$\frac{\partial f(x,y)}{\partial y} = \frac{\partial}{\partial y}(e^x \sin y + e^y \cos x)$$

$$\frac{\partial f(x,y)}{\partial y} = e^x(1)(\cos y) + e^y(1)\cos x$$

$$\frac{\partial f(x,y)}{\partial y} = e^x \cos y + e^y \cos x \quad \text{(i)}$$

$$\frac{\partial^2 f(x,y)}{\partial y^2} = e^x(-\sin y)(1) + e^y(1)\cos x$$

$$\frac{\partial^2 f(x,y)}{\partial y^2} = -e^x \sin y + e^y \cos x \quad \text{(ii)}$$

Laplace eqn $\frac{\partial f(x,y)}{\partial x^2} + \frac{\partial f(x,y)}{\partial y^2} = 0$

$$e^x \sin y - e^y \cos x - e^x \sin y + e^y \cos x = 0$$

$$0 = 0$$

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Page # 05

Q4

Q4 $a = i + 2j - k$ $b = -2i + 3k$, $c = 7j - 4k$

Find $a \cdot (b \times c)$

Solution

$$a \cdot (b \times c) = \begin{vmatrix} 1 & 2 & -1 \\ -2 & 0 & 3 \\ 0 & 7 & -4 \end{vmatrix}$$

$= 1[0 \times -4 - 7 \times 3] - 2[-2 \times -4 - 3 \times 0]$

$-1[0 \times 0 - (-2)(7)$

$= 1(-21) - 2(-8) - 1(14)$

$= -21 + 16 - 14$

$= -35 + 16$

$= -19$

~~$a \cdot (b \times c)$~~

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Project 06

Q5

Q 5

$4x^2 - y^2 - 3z^2 = 16 \quad P(2, -3, 1)$

Solution:-

$4x^2 - y^2 - 3z^2 = 16$

$f(x, y, z) = 4x^2 - y^2 - 3z^2 = 16$

$\Delta f = 8x\,i - 2y\,j - 6z\,k = 16$

$P_0(2, -3, 1)$

$\Delta f = (2, -3, 1) = 16i + 6j - 6k$

tangent plan on point P_0.

$16(x - 2) + 6(y + 3) - 6(z - 1) = 16$

Normal line

$x = 1 + 2t$

$y = 2 + 4t$

$z = 4 + 4t$