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Section B

Subject Diffiontal

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Assigment 02

Dated 14 - June 2020

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Question # 01

1: $x^3 y''' + 2x^2 y' + 2y - 10x + \frac{10}{x}$

Solution:-

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3 D^3 y + 2x^2 D^2 + 2y = 10x + 10x^{-1}$$

$$(x^3 D^3 + 2x^3 D + 8)y = 10x + 10x^{-1} \quad (1)$$

Let $x = e^t \Rightarrow t = \ln x$

$$xD = D$$
$$x^2 D^2 = D(\Delta - 1) = \Delta^2 - \Delta$$
$$x^3 D^3 = \Delta(\Delta - 1)(\Delta - 2)$$

Substituting into eq (1)

$$\Delta - 3\Delta^2 + 2\Delta + 2(\Delta^2 - \Delta) + 2)y = 10^x + 10^{x^{-1}}$$

$$(\Delta^3 - \Delta^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = 10e^x + \frac{10}{e^t}$$

Using Synthetic division

-1	1	-1	0	2
		-1	2	-2
	1	-2	2	$\boxed{0}$

$$\Delta^2 - 2\Delta + 2 = 0$$

Now

Using quadratic formula

$a = 1$, $b = -2$, $c = 2$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-(-2) \pm \sqrt{-2^2 - 4(1)(2)}}{2a}$$

$$D = \frac{2 \pm \sqrt{4-8}}{2}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = 2 \pm \sqrt{-1} \times \sqrt{4}$$

$$D = \frac{2 + 2i}{2}$$

$$D = \frac{\cancel{2}(1 + i)}{\cancel{2}}$$

$$D = 1 \pm i$$

Since roots are complex

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Now particular integration

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot 10/e^t$$

$$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^3 - (1)^2 + 2}$$

$$= \frac{\cancel{10}^{5} e^t}{2} + \frac{\cancel{10}^{5} e^{-t}}{2}$$

$$= 5e^t = 5e^{-t}$$

$$y_p = 5e^t + 5e^{-t}$$

General Solution

$$y = y_c + y_p$$

$$y = e^{-x}(c \cos t + 2 \sin t) + 5e^t + 5e^{-t}$$

put $e^t = x$ and $t = \ln x$

$$y = e^{-x}(c_1 \ln x + c_2 \sin/x + 5e^x + 5e^{-x}$$

QUESTION # 02 :-

2) $x^3\frac{d^3y}{dx} + 4x^2\frac{d^2y}{dx} - 5x\frac{dy}{dx} - 15y = x^4$

Solution:-

Let $\frac{d}{dx} = D$

$x^3 D^3y + 4x^2D^2y - 5xDy - 15y = x^4$

$x^3D^3 + 4x^2D^2 - 5xD - 15)\, y = x^4$

Let

$x = e^t \Rightarrow t =$

$xD = \Delta$

$x^2D^2 = D(D-1) = \Delta^2 - \Delta$

$x^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta^2 + 2\Delta$

Now Subtituting

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$

$(\Delta^3 - 3\Delta^2 + 2\Delta + 4(\Delta^2 - \Delta) - 5(\Delta) - 15)y = e^{4t}$

$(\Delta^3 + \Delta^3 - 7\Delta - 13)y = e^{4t}$

Synthetic division.

5	1	+1	-7	-15
		3	12	15
	1	4	5	0

$D^2 + 4D + 5 = 0$

cladratic Formula

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$$

$$D = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$D_1 = \cancel{2}\left(\frac{-2 \pm 2}{\cancel{2}}\right)$$

$$y_c = e^{u-1}\left(c_1 \cos t + 2 \sin t\right)$$

For $yp = ?$

$$yp = \frac{1}{D^3 + D^2 - 7D - 15} e^{3t}$$

$$= \frac{1}{(4)^3 + (4^2) - 7(4) - 15} e^{4t}$$

$$= \frac{1}{64 + 16 - 28 - 15} e^{4t}$$

$$= \frac{1}{} e^{4t}$$

$y_p = \frac{1}{37} e^{4t}$

hence

$$y = y_c + y_p$$

$$y = (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$$

again put $t = \ln x$ and $x = \ln x$

$$y = e^{3t} (c_1 \cos \ln x + c_2 \sin \log x + \frac{1}{37} e^{4t}$$

Ans:-

Question # 03 :-

$$x^3 y'' + 2x y' - 6y = 10x^2$$

Solution :-

$$y(1) = 1 \quad \text{and} \quad y(1) = -b$$

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 6y = 10x^2$$

$$\Rightarrow \left(x^2 \frac{d^2}{dx^2} + 2x^2 \frac{d}{dx} - 6\right) y = 10x^2$$

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put $xD = D \Rightarrow x^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$

$$x = e^t \quad \text{and} \quad \log x = t$$

$$(\Delta^2 - \Delta + 2\Delta - 6) y = 10e^{2t}$$

$$(\Delta^2 + \Delta - 6) y = 10e^{2t}$$

The characteristic equation

$$\Delta^2 + \Delta - 6 = 0$$

$$\Delta^2 + 3\Delta - 2\Delta - 6 = 0$$

$$\Rightarrow \Delta(\Delta + 3) - 2(\Delta + 3) = 0$$

$$\Rightarrow (\Delta + 3)(\Delta - 2) = 0$$

$\Delta = 2$, $\Delta = -3$

Since root are real and distinct

for $y = ?$

$y_c = c_1 e^{-3t} + c_2 e^{2t}$

for $y_p = ?$

$y_p = \frac{1}{\Delta^2 - \Delta - 6} \cdot 10^{2t}$

$= \frac{10}{\Delta^3 - \Delta - 6} e^{2t}$

$= 10 \frac{1}{0} e^{2t}$ fails

Now :-

$10 \frac{1}{d/d\Delta\,(\Delta^2 + \Delta - 6)} e^{2t}$

$\Rightarrow 10 \frac{t}{2\Delta + 1} e^{2t}$

$= 10 \frac{1 \cdot t}{4+1} e^{2t}$

$y_p = 2t\, e^{2t}$

General Solution:-

(9)

$y = y_c + y_p$

$= c_1 e^{-3t} + c_2 e^{2t} + 2te^{2t}$

$y = c_1 x^{-3} + (2x^2 + 2(\log x) x^2$ —— (B)

Put $y(1) = 1$ i-e $x = 1$, $y = 1$ in (B)

$1 \quad c_1 (1)^{-3} + 2(1)^2 + 2\log(1)$

$1 = c_1 + c_2 \longrightarrow$ (C)

Now differential eq (B) w.r.t x

$y' = 3c_1 x^{-4} + 2(2x + \frac{2}{x}(x^2) + 4x \log x$

Now put $y'(1) = -6$ i-e $y' = -6$ and $x = -6$

$-6 = -3c_1 + 2(2 + 2 + 0$

$\Rightarrow -6 = -3c_1 + 2(2+2)$

$\Rightarrow -6 - 2 = 3c_1 + 2(2+2)$

$-8 = -3c_1 + 2c_2 \longrightarrow$ (1)

x ing eq (1) with (2) and ing from (2)

$2c_1 + 2c_2 = 2$

$\underline{3c_1 \pm 2c_2 = \mp 8}$

$5c_1 \quad = 10$

$-8 = -6 + 2c_2$

$2(2) = -8 + 6$

$2c_2 = -2$

$c_2 = \frac{-\cancel{2}\ 1}{\cancel{2}}$

$c_2 = -1$

Now put the value of c_1 and c_2 in eq (B)

$y = 2x^{-3} - x^2 + 2(un) x(x^2)$

QUESTION # 04 :-

$$x^2y'' + 7xy' + 5y = x^5$$

$$y(0) = 2 \text{ and } y'(1) = 2$$

Solution :

$$x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5$$

$$\Rightarrow \left(x^2\frac{d^2}{dx^2} + 7x\frac{d}{dx} + 5\right)y = x^5 \quad \text{—— (A)}$$

Put $xD = \Delta \Rightarrow x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$

$x = e^t \Rightarrow \log x = t$ in eq (2)

$$\Rightarrow (\Delta^2 - \Delta + 7D + 5)y = e^{5t}$$

$$\Rightarrow (\Delta^2 + 6D + 5)y = e^{5t}$$

$$= \frac{-6 \pm \sqrt{36-20}}{2}$$

$$= \frac{-6 \pm \sqrt{16}}{2}$$

$$= \frac{-6 \pm \sqrt{4^2}}{2}$$

$$= \frac{\cancel{2}(-3 \pm 2)}{\cancel{2}}$$

$D = -3 \pm 2$. Since roots are equal and distict.

$y_c = c_1 e^{-5t} + c_2 e^{-t}$

For $y_P = ?$

$$y_P = \frac{1}{D^2+6D+5} e^{5t}$$

$$= \frac{1}{(5)^2+6(5)+5} e^{5t}$$

$$= \frac{1}{60} e^{5t}$$

Now general Solution is

$y = y_c + y_P$

$y = c_1 e^{-5t} + c_2 e^{-t} + \frac{1}{60} e^{5t}$

$y = c_1 e^{-5t} + 2x^{-1} + \frac{1}{60} x^5 \longrightarrow (B)$

$x = 0$ put in this equation

No in eq (B) $e^0 = 1$

put $y(0) = 2$ i-e $y = 2$ and $x = 2$

$2 = c_1 (2)^{-5} + c_2 (2)^{-1} + \frac{1}{60} (2)^5$

$2 = -32 c_1 - 2(\,) + \frac{1}{\cancel{60}_{15}} (\cancel{32}^{8})$

$2 = -32 c_1 - 2c_2 + 8/15$

$2 = \frac{-8}{15} = -32 c_1 - 2c_2$

Now differentiation eq (B) w.r.t (x)

$y' = -5c_1 x^{-6} - (2x^{-2} + \frac{1}{12} x^4$ —

Put $y(1) = 2$ i-e $y' = 2$ and $x = 2$

in above equation

(14)

$2 = -5c_1 (2)^{-6} - (2)(25)^2 + \frac{1}{12}(2)$

$2 = -5c_1(-64) - c_2(4) + \frac{1}{12}(16)$

$2 = 32c_1 + 4c_2 + 4/3$

$\Rightarrow 2 - \frac{4}{3} = 320c_1 + 4c_2$

$\Rightarrow 2/3 = 320c_1 + 4c_2$ —— (D)

x ing eq (c) with and then —ing eq from

$\frac{-44}{15} = 64c_1 + 4c_2$

$\frac{-44}{15} = 64c_1 + 4c_1$

$\pm\ 2/3 = \pm 320c_1 \pm 4c_2$

$\frac{34}{15} = -256c_1$

$c_1 = \frac{34}{15} \times 256$

$\boxed{c_1 = 580}$

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put the value of c_1 in eq c

$$\frac{2}{15} = -32(580) - 2C_2$$

$$\Rightarrow \frac{22}{15} = -18560 - 2C_2$$

$$\Rightarrow \frac{22}{15} + 18560 = -2C_2$$

$$\Rightarrow \frac{18561}{-2} = C_2$$

$$\Rightarrow \frac{-18561}{-2} = C_2$$

$$\boxed{-9280 = C_2}$$

Now put the value of 1 and 2 in eq (B)

$$y = 580x^{-5} - 9280x^{-1} + \frac{1}{60}x^5$$

$$y = \frac{580}{x^3} - \frac{9280}{x} - \frac{1}{60}x^5$$

Ans: