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ID#12998
Subject: Linear Algebra
Program: BS(SE)

Question No. 1 :

Solution:

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Q1:-

Sol: let $[A - \lambda I] = 0$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -17 & 8 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{vmatrix} = 0$$

$$-\lambda \begin{vmatrix} -\lambda & 1 \\ -17 & 8-1 \end{vmatrix} - 1 \begin{vmatrix} 0 & 1 \\ 4 & 8-\lambda \end{vmatrix} = 0$$

$$-\lambda[-\lambda(8-\lambda) + 17] - 1(-4) = 0$$

$$-\lambda[-8\lambda + \lambda^2 + 17] + 4 = 0$$

$$-\lambda^3 + 8\lambda^2 - 17\lambda + 4 = 0$$

$$\lambda^3 - 8\lambda^2 + 17\lambda - 4 = 0$$

$\lambda = 4$ is our root,

So,

$$\begin{array}{c|cccc} 4 & 1 & -8 & 17 & -4 \\ & \downarrow & 4 & -16 & 4 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$

$$\lambda^2 - 4\lambda + 1 = 0$$

$$\lambda = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2}$$

Here,

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

$$\boxed{\lambda = 2 \pm \sqrt{3}}$$ Ans

Question No. 2
Solution:

Question 2:

Sol: A can be diagnalized if there exists an invertible matrix P and diagnol matrix D such that $A = PDP^{-1}$,

Here now,

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

Step 1: Find eigenvaluos of the matrix A,

$|A - \lambda I| = 0$

$$\begin{vmatrix} (-\lambda) & 0 & -2 \\ 1 & (2-\lambda) & 1 \\ 1 & 0 & (3-\lambda) \end{vmatrix} = 0$$

$$(-\lambda)((2-\lambda)\times(3-\lambda) - 1\times 0)\,(1\times(3-\lambda) - 1\times 1) + (-2)(1\times 0 - (2-\lambda)\times 1) = 0$$

$$(-\lambda)\left(((6-5\lambda+\lambda^2)-0\right) - 0((3-\lambda)-1) - 2(0-(2-\lambda)) = 0$$

$$(-\lambda)(6-5\lambda+\lambda^2) - 0(2-\lambda) - 2(-2+\lambda) = 0$$

$$(-6\lambda + 5\lambda^2 - \lambda^3) - 0 - (-4+2\lambda) = 0$$

$$(-\lambda^3 + 5\lambda^2 - 8\lambda + 4) = 0$$

$$-(\lambda-1)(\lambda-2)(\lambda-2) = 0$$

$(\lambda-1) = 0$ or $(\lambda-2) = 0$ or $(\lambda-2) = 0$

→

Cont....

The eigenvalues of the matrix A are given by
$\lambda = 1, 2,$

Step 2,

⇒ Find eigenvectors for $\lambda=2$

$$v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Step 3,

The eigenvectors compose the Column of matrix P

$$P = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Step 4,

The diagnol matrix D is composed of the eigenvalues,

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Step 5,

Now find P^{-1}

$$|P| = \begin{vmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= -2 \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 0 \times \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - 1 \times \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

Cont....

$= -2\times(1\times1-0\times0)+0\times(1\times1-0\times1)-1\times(1\times0-1\times1)$

$= -2\times(1+0)+0\times(1+0)-1\times(0-1)$

$= -2\times(1)+0\times(1)-1\times(-1)$

$= -2+0+1$

$\boxed{= -1}$ Ans:

Question No. 3

Solution:

Question 3:

Sol

$A = (1, -2, 3)$, $B = (5, 6, -1)$, $C = (3, 2, 1)$

$$|D| = \begin{vmatrix} 1 & -2 & 3 \\ 5 & 6 & -1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= 1 \times \begin{vmatrix} 6 & -1 \\ 2 & 1 \end{vmatrix} + 2 \times \begin{vmatrix} 5 & -1 \\ 3 & 1 \end{vmatrix} + 3 \times \begin{vmatrix} 5 & 6 \\ 3 & 2 \end{vmatrix}$$

$$= 1 \times (6 \times 1 - (-1) \times 2) + 2 \times (5 \times 1 - (-1) \times 3) + 3 \times (5 \times 2 - 6 \times 3)$$

$$= 1 \times (6 + 2) + 2 \times (5 + 3) + 3 \times (10 - 18)$$

$$= 1 \times (8) + 2 \times (8) + 3 \times (-8)$$

$$= 8 + 16 - 24$$

$$= 0$$

Since $|D| = 0$, so vectors A, B and C are linearly dependent.

Question No .4

Part (a)

Ans:

Q4:-
Part (a):
Sol: : The set $P_n(x)$ of all polynomials over $\mathbb{R}$ in variable x of degree $\leq n$ forms a vector space over $\mathbb{R}$.

if $f_n(x) = a_0 + a_1x + \dots\dots + a_nx^n$
and $g_n(x) = b_0 + b_1x + \dots\dots + b_nx^n$, $a_i, b_i \in \mathbb{R}$.

Then, $f_n(x) + g_n(x) = (a_0 + b_0) + (a_i + b_i)x + \dots\dots + (a_n+b_n)x^n$
is a Polynomial of $P_n(x)$

The associative additive property is induced from the additive association property of $\mathbb{R}$.

The zero polynomial $f_n(x) = 0$ of degree zero acts as the additive identity,

and $-f_n(x) = -a_0 + (-a)x + \dots\dots + (-a_n)x^n$ is the additive inverse of $f_n(x)$.

Commutative property follows from the Commutative property of $\mathbb{R}$. Hence $P_n(x)$ is an additive abelian group.

The scalar multiplication of $a \in \mathbb{R}$ by $f_n(x)$ is defined by,

$a \cdot f_n(x) = aa_0 + (aa_1)x + (aa_2)x_2 \dots\dots + (aa_n)x^n \in P_n(x)$

Part (b):

Ans:

Question 4:

Part (b):

Ans:- Let F be a field. A non empty set V togather with two binary operations (+) and (·) forms the Algebric structure of a vector space over the field F, if;

(a)= V forms an additive (+ve) abelian group.

(b) = The Scaler multiplication (·) as a function form $F \times V$ into V observes the following Properties, which are given below;

(1): $\forall\, \alpha \in F,\ x, y \in V,\ \alpha.(x+y) = \alpha x + \alpha y$

(2): $\forall\, \alpha, \beta \in F,\ (\alpha+\beta)\cdot x = \alpha x + \beta x,\ \forall x \in V.$

(3): $\forall\, \alpha, \beta \in F,\ \forall x \in V;\ \alpha(\beta x) = (\alpha\beta)x$

(4): For e_F, the identity of F, $e_F \cdot x = x,\ \forall x \in V.$