Iqra National University, Peshawar
Department of Electrical Engineering

INU
IQRA NATIONAL UNIVERSITY

Assignment
Date:19/8/2020

Course Code:	MTH 102			Course Title:	Calculus and analytic geometry
Prerequisite:				Instructor:	HIMAYATULLAH
Module:	3	Program:	BEE	Total Marks:	30

Q1.	(a)	. **Identify** $\lim_{x\to 1}\frac{x^2+x-2}{x^2-x}$	Marks 5 CLO1 C1
	(b)	Find the first order derivatives of the function $y=(3-x^2)(x^3-x+1)$	Marks 5 CLO1 C1
Q2	(a)	. At a time t the body position along s-axise is given by $s=t^3-t^2+9t$ (i) Find the body acceleration each time when the velocity is zero (ii) Find the body speed each time when the acceleration is zero.	Marks 10 CLO2 C2
Q3	(a)	Find the equation of tangent and normal to the curve at the given point Where $x^2-xy+y^2=7$, $(2, 3)$	Marks 10 CLO1 C1

Name : Naveed Ali

ID No : 16753

Subject : Calculus & Analytic geometry

Q1 Identify

(a) $$\lim \frac{x^2+x-2}{x^2-x}$$

Sol:.

$$\text{Lim} \frac{x^2+x-2}{x^2-x}$$

Direct putting $x=1$

$$\Rightarrow \frac{(1)^2+(1)-2}{(1)^2-1}$$

$$= \frac{1+1-2}{1-1} = \frac{2-2}{0} = 0/0$$

undefine.

By factrization

x^2+x-2

$x^2+2x-x-2$

$-2x$ $+2x$ $-x$ x

$x(x+2) - 1\,(x-2)$

$(x-1)\,(x+2$

Now

$$x^2+x-2 = (x-1)(x+2)$$

$$\lim_{x\to 1} \frac{x^2+x-2}{x^2-x}$$

$$\lim_{x\to 1} \frac{(x-1)(x+2)}{x^2-x}$$

$$\lim_{x\to 1} \frac{\cancel{(x-1)}(x+2)}{x\,\cancel{(x-1)}}$$

$$\lim_{x\to 1} \frac{x+2}{x}$$

Applying Limit

$$\frac{1+2}{1}$$

$= 3/2$ Ans

Q2 Find the first derivatives of
Part B the function

$$y = (3-x^2)(x^2-x+1).$$

Sol: First order derivate

$$y' = dy/dx = ?$$

$$dy/dx = d/dx\, (3-x^2)(x^2-x+1)$$

using product rule

$$dy/dx = d/dx.$$

$$dy/dx = (3-x)\, d/dx\, (x^2-x+1)$$

$$+ (x^2-x-1)\, d/dx\, (3-x)$$

$$= (3-x)\; d/dx\,(x^2) - \cancel{d/dx\,(x)}$$
$$+ d/dx\,(1) + (x^2-x-1)$$

$$(d/dx\,(3) - d/dx\,(x))$$

$$\Rightarrow (3-x)(2x-1-0) +$$
$$(x^2-x+1)(0-1)$$

$= (3-x)(2x-1) + (x^2-2+1)(-1)$

$6x-3-2x^2 + (-x^2+x-1)$

$6x-3-2x^2-x^2+x-1$

$6x+x-2x^2-x^2-3-1$

$-3x^2+7x-4$

Ans

— x — x — x — x —

Q3. Find the equation of tangent and normal to the curve at the given point

Where $x^2-xy+y^2=7$. $(2,3)$.

Sol:- $x^2-xy+y^2=7$ $(2,3)$

Equation of Tangent $\Rightarrow$ $y-y_1 = f'(x)(x-x_1)$

Equation of Normal $\Rightarrow$ $y-y_1 - \frac{1}{f'(x)}(x-x_1)$

where $(2,3) = (x_1, y_1)$

So

$x_1 = 2$

$y_1 = 3$

1st find $f'(x) = dy/dx$

Taking derivate both sides

$$d/dx\,(x^2 - xy + y^2) = d/dx\,(7)$$

$$d/dx\,(x^2) - d/dx\,(x,y) + d/dx\,(y^2) = d/d\,(7)$$

$$2x - [x\,dy/dx + y\,dx/dx] + 2y\,dy/dx = 0$$

$$2x - x\,dy/dx - y + 2y\,dy/dx = 0$$

$$2y\,dy/dx - x\,dy/dx + 2x - y = 0$$

$$dy/dx\,[2y - x] = -2x + y$$

$$f'(x) = \boxed{dy/dx = \frac{-2x+y}{2y-x}}$$

Now slope

$$\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{-2x+y}{2y-x}$$

put $x=2$

$y=3$

$$= \frac{(2)(2)+3}{2(3)-2} = \frac{-4+3}{6-2} = -1/4$$

$$\boxed{\text{Slope} = -1/4}$$

Now putting value in equation of tangent

$f'(x) = m = \text{slope} = -1/4$

$x1 = 2$

$y1 = 3$

$y - y_1 = f'(x)(x - x_1)$

$y - 3 = -1/4\ (x-2)$

~~$y-3=-x$~~ $y - 3 = -\frac{(x-2)}{4}$

$4y - 12 = -x + 2$

$4y + x - 12 + 2 = 0$

$$\boxed{4y + x - 10 = 0}$$

equation of tangent

Now equation of Normal.

$$y - y_1 = -1/f'(x) \quad (x - x_1)$$

$$y - 3 = -1/(-1/4) \quad (x - 2)$$

$$y - 3 = (-1) \div (-1/4) \quad (x-2)$$

$$y - 3 = -1 \times (-4) \quad (x - 2)$$

$$y - 3 = 4x - 8$$

$$\cancel{y - 3 = 4x - 3 + 8 = 0}$$

$$\cancel{y - 3 = 4x}$$

$$\cancel{y - 3 - 4}$$

$$y - 4x - 3 + 8 = 0$$

$$\boxed{y - 4x + 5 = 0}$$

Equation of Normal.

Q2: ~~Fin~~ At a time t the body
(a) position along s-axise is given
by $s = t^3 - t^2 + 9t$

Sol: Find velocity & acceleration 1st

As we know that
$$v = ds/dt$$

$$\& \quad a = dv/dt$$

so
$$v = d/dt\,[s)$$
$$v = d/dt\,[t^3 - t^2 + 9t)$$

$$v = d/dt\,[t^3] - d/dt\,\{t^2\} + d/dt\,(9)$$

$$\boxed{v = 3t^2 - 2t + 9}$$

$$a = dv/dt \qquad a = d/dt\,[3t^2 - 2t + 9)$$

$$a = d/dt\,3t^2 - d/dt\,(2t) + d/dt\,(9)$$

$$a = 6t - 2 + 0$$
$$a = 6t - 2 + 0$$
$$\boxed{a = 6t - 2}.$$

(b) Now find the speed when acceleration is zero.

So put $a = 0$

$0 = 6t - 2$

$2 = 6t$

$6t = 2$

$t = \cancel{2}/\cancel{6}$ $\boxed{t = 1/3 \text{ sec}}$

Now speed

$v = 3t^2 - 2t + 9$

put $t = 1/3$

$v = 3(1/3)^2 - 2(1/3) + 9$

$v = 3/9 - 2/3 + 9$

$v = 1/3 - 2/3 + 9$

$v = \frac{1 - 2 + 27}{3}$

$v = 1 + 25/3$

$v = 26/3$

$\boxed{v = 8.6 \text{ m/s}}$

(a) Find the body Post acceleration each time when the velocity is zero.

So put $v=0$

$$v = 3t^2 - 2t + 9$$

$$0 = 3t^2 - 2t + 9.$$

Using Quadratic formula.

$a = 3,\quad b = -2 \quad c = 9$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

using Q. equation putting value.

$$t = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(9)}}{2(3)}$$

$$t = \frac{2 \pm \sqrt{4 - 108}}{6}$$

$$t = \frac{2 \pm \sqrt{-104}}{6}$$

$t = \frac{2 \pm \sqrt{104}\, i}{6} = \frac{2 \pm 10.1 j}{6}$

$t = \frac{\cancel{2}(1+5j)}{\cancel{6}}$

$t_1 = \frac{1+5j}{3}$

$t_2 = \frac{1-5j}{3}$

Now accelerat.

$a_1 = 6t - 2$

$a_1 = \cancel{6}\left(\frac{1+5j}{\cancel{3}}\right) - 2$

$a_1 = 2 + 5j - 2$

$\boxed{a_1 = 0 + 5j \ m/s}$

$a_2 = \cancel{6}\left(\frac{1-5j}{\cancel{3}}\right) - 2$

$a_2 = \cancel{2}\ 05 - j - \cancel{2}$

$\boxed{a_2 = -5j \ m/s}$

The End.