(1)

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Subject # multivariate calculus

Mid-term

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Q1) If $(x+yi)/i = (7+9i)$, when x and y are real, what is the value of $(x+yi)(x-yi)$?

Solution:-

$= (x+yi)/i = (7+9i)$

$= (x+yi) = i(7+9i)$

$= (x+yi) = 7i - 9$

$= (x+yi)(x-yi) = (-9+7i)(-9-7i)$

$= 81 + 94 = 130$

(Q2) Find the values of x and y and in the following equation, give further that $x \in R$, $y \in R$.

$(x+iy)(2+i) = 3-i$

Solution:-

$(x+iy)+(2+i) = 3-i$

$(x+iy) = \left(\frac{3-i}{2+3i}\right)\left(\frac{2-3i}{2-3i}\right)$

$= \frac{6-9i-2i+3i^2}{4-9i^2}$

$i^2 = -1$
$-9i^2 = 9$

$= \frac{9+11i}{13}$

$= \frac{9}{13} + \frac{11}{13}i$

$x = \frac{9}{13}$, $y = -\frac{11}{13}$

Q3)

Differeniate

i) $f(x) = (\ln x)^4$

$\frac{dy}{dx} = \frac{4(\ln x)^3}{x}$

differeniate using the chain rule

given $y = f(g(x))$ then

$\frac{dy}{dx} = f'(g(x)) \times g'(x)$ ← chain rule

$y = (\ln x)^4$

$\Rightarrow \frac{dy}{dx} = 4(\ln x)^3 \times \frac{d}{dx}(\ln x)$

$= \frac{4(\ln x)^3}{x}$

ii) $g(x) = x^2 . \ln x$

Solution:-

$\boxed{g''(x) = 2\ln(x) + 3}$

By the product rule we get.

$g'(x) = 2x\ln(x) + x^2 . 1/x$

Simplifying

$f'(x) = 2x\ln(x) + x$

$f''(x) = 2\ln(x) + 2x . \frac{1}{x} + 1$

Simplyfying we get

$f''(x) = 2\ln(x) + 3$

Q4) Solve the equation

$2z^2 - 2iz - 5 = 0, z \in C.$

Solution

$= 2z^2 - 2iz - 5 = 0$

$= 2(x+yi)^2 - 2i(x+yi) - 5 = 0$

= Expand $2(x+yi)^2 - 2i(x+yi)$

$-5x \quad (2x^2 - 2y^2 + 2y - 5) + i(-2x$

$+ 4xy)$

$= (2x^2 - 2y^2 + 2y - 5) + i(-2x + 4xy) = 0$

$= (2x^2 - 2y^2 + 2y - 5) + i(-2x + 4xy)$

$= 0 + 0i.$

Q5) Express $4-\sqrt{5}i$ in polar form.

Solution:-

$4-\sqrt{5}i$

$= 4-\sqrt{5}i$ in $= 4-\sqrt{5}$ in $\sqrt{i}$

Steps

$4-\sqrt{5}$ in

$= \sqrt{5i} = \sqrt{5}\sqrt{i}$

$= 4-\sqrt{5}$ in $\sqrt{i}$

Q6) Find the limit

$$\lim_{z \to 8} \frac{2z^2 - 17z + 8}{8 - z}$$

Solution.-

$$\lim_{z \to 8} \left(\frac{8z^2 - 17z + 8}{8 - z} \right) = -15$$

$$\lim_{z \to 8} \left(\frac{2z^2 - 17z + 8}{8 - z} \right)$$

Simplify

$$= \frac{2z^2 - 17z + 8}{8 - 2} : -2z + 1$$

$$= \lim_{z \to 8} (-2z + 1)$$

$$z = 8$$

$$= -2 \quad 8 + 1$$

Simplify $= -15$