NAME Shaheryar

ID 7891

Section A

Assigment #3

Subject Differential

Application of Partial differential equation:-

Many engineering Problems are governed by differents types of Partial differential equations, and some of the more important types are given belwo.

Tricomi equation:-

$$y \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \begin{cases} y > 0 : \text{elliptic} \\ y < 0 : \text{hyperbolic} \end{cases}$$

Laplace equation (or variants): $\partial 2 \varphi \, \partial x 2 + \partial 2 \varphi \, \partial y 2 = \nabla 2 \varphi = 0$

Poisson's equation:-

$$\partial 2 \varphi \, \partial x 2 + \partial 2 \varphi \, \partial y 2 = f(x, y)$$

Helmholtz equation: $\partial 2 \varphi \, \partial x 2 + \partial 2 \varphi \, \partial y 2 + C 2 \varphi = 0$

Plate bending:-

$$\nabla 2 \, \nabla 2 w = \nabla 4 w = q D$$

(2)

Wave equation:-
$$\frac{\partial^2 u}{\partial t^2} - c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$$

Fourier equation:- $\partial T \partial t = a\ (\partial_2 T \partial x 2)$

Separable differential equations:-

For equations which can be expressed in separable form as shown below, the solution can be obtained easily as,

$$\frac{dy}{dx} = F(x,y) \quad \frac{dy}{\phi(y)} = f(x)\, dx \quad \int \frac{dy}{\phi(y)} = \int f(x)\, dx + C$$

$$M(x,y)\, dx + N(x,y)\, dy = 0 \quad M(x)\, dx = N(y)\, dy$$

then $\int M(x)\, dx = -\int N(y)\, dy + C.$

3)

Separable differential equations :-

For equations which can be expressed in separable from as shown below, the solution can be obtained easily as.

$$\frac{dy}{dx} = f(x, y) \quad \frac{dy}{\pi(y)} = f(x)\,dx \quad \int \frac{dy}{\pi(y)} = \int f(x)\,dx + C$$

$$M(x, y)\,dx + N(x, y)\,dy = 0 \quad M(x)\,dx = -N(y)\,dy$$

then $\int M(x)\,dx = -\int N(y)\,dy + C$

Example:-

$$\frac{dy}{dx} = \cancel{8}x^3+(y^2+1) \Rightarrow \frac{dy}{y^2+1} = x^3\,dx$$

$$\int \frac{dy}{y^2+1} = \int x^3\,dx + C \Rightarrow \tan^{-1} y = \frac{1}{4} x^4 + C$$

$$\Rightarrow y = \tan\left(\frac{1}{4} x^4 + c\right)$$

Example:-

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)} \quad \text{subject to } y(0) = -1$$

Since is a separable function, the problem can be solved as

$$2(y-1)\,dy = (3x^2+4x+2)\,dx$$

$$y^2 - 2y = x^3 + 2x^2 + 2x + c$$

Example:

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)} \quad \text{subject to } y(0) = -$$

Since this is a separable function, the problem can be solved as

$$2(y-1)\,dy = (3x^2+4x+2)\,dx$$

$$y^2 - 2y = x^3 + 2x^2 + 2x + c$$

Based on the boundary condition, $C = 3$,

hence $y^2 - 2y = x^3 + 2x^2 + 2x + 3$.

This quadratic equation in y^2 can be solved with two solutions by the quadratic equation as:

[illegible]

$y = 1 \sqrt{x^3 + 2x^2 + 2x + 4}$ and $y = 1 + \sqrt{x^3 + 2x^2 + 2x + 4}$

Since the second solution does not satisfy the boundary condition, it will not be accepted, hence, the solution to this differential equation is obtained

Variation of Parameters:-

for the following equation form, it is possible to solve it by variation of parameters.

for $$\frac{dy}{dx} = P(x)\, y + Q(x)$$

put $y = C(x)\, e^{\int P(x)\, dx}$ by differentiating, it givens.

$$\frac{dy}{dx} = \frac{dC(x)}{dx} e^{\int P(x)\, dx} + \underbrace{C(x)\, P(x)\, e^{\int P(x)}}_{P(x)\, y}$$

Substitute it to the original ODE

$$\frac{dc(x)}{dx} = Q(x)\, e^{-\int P(x)\,dx}$$ Computing it

form, it gives

$$C(x) = \int Q(x)\, e^{-\int P(x)\,dx}\, dx + C$$

Example:-

$$(x+1)\frac{dy}{dx} - ny = e^{x}(x+1)^{n+1}$$

This equation is now expressed as.

$$\frac{dy}{dx} = P(x)\, y + Q(x)$$

$$\frac{dy}{dx} = \frac{n}{x+1}\, y + \underbrace{e^{x}(x+1)^{n}}_{Q(x)}$$

for $x \neq -1$

Solving the homogeneous part of the ODE

$$\frac{dy}{dx} = \frac{n}{x+1}\, y \quad \text{then} \quad \frac{dy}{y} = \frac{n}{x+1}\, dx$$

$$\ln|y| = \ln(n)|x+1| + c_1$$

$$y = c\,(x+1)^{n}$$

1)

Look for solution $y = c(x)(x+1)^n$

where $c(x)$ is the variation of parameter

Substitute it to the ODE

$$\frac{dc(x)}{dx}(x+1)^n + nc(x)(x+1)^{n-1} = nc(x)$$

$$(x+1)^{n-1} = nc(x)(x+1)^{n-1} + e^x (x+1)^n$$

$$\frac{dy}{dx} = \frac{n}{x+1} y + e^x (x+1)^n$$

Comparison givens $\frac{dc(x)}{dx} = e^x$

Integration of this equation gives.

$$C(x) = e^x + \bar{C}$$

General Solution is hence given by

$$y = (x+1)^n (e^x + \bar{C})$$

5)

The Bernoulli equation is an important equation type which can be solved in a similar way by variation of parameters. Consider the following form of equation.

$$\frac{dy}{dx} = P(x)\,y + Q(x)\,y^n$$

step 1:- put $z = y^{1-n}$

step 2:- Then $\frac{dz}{dx} = (1-n)\,y^{-n}\,\frac{dx}{dy}$

$$\frac{dz}{dx} = (1-n)\,P(x)\,z + (1-n)\,Q(x)$$

The non linear ODE now becomes linear ODE. It can be solved by formula.

step 3:- $n = -1,\ z = y^2$. Inverting z to get y

$$\frac{dy}{dx} = \frac{y}{2x} + \frac{x^2}{2y}$$

$$\frac{dz}{dx} = \frac{1}{x}z + x^2$$

9)

$$z = e^{\int \frac{1}{x} dx} \left(\int x^2 e^{-\int \frac{1}{x} dx} dx + c \right) = cx + \frac{1}{2} x^3$$

Back substitution of $z = y^2$ of $z = y^2$

$$y^2 = cx + \frac{1}{2} x^3$$

Homogeneous equations:-

For equation of the following type, where all the Coefficients are Constant, it Can be evaluated according to different.

La place equation.

La place equation forms an important governing Condition for many types of Problems. Some of the more Common forms are given by.

Three dimensional La place equation:-

$$u_{xx} + u_{yy} + u_{zz} = 0$$

a)

Two dimensional heat Conduction:-

$$a^2 (u_{xx} + u_{yy})\ u_t$$

Two dimensional Seepage Problem.

$$(K_x\ u_{xx} + K_y\ u_{yy}) = 0$$

There a two major types of boundary Conditions to this problem.

Dirichlet Problem:-

Boundary Conditions prescribed as u.