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Paper : Differential Equation

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Q No. 1

Solve The Initial value Problem.

$$dy/dt = e^{y-t} \sec(y)(1+t^2) \quad y(0)=0$$

Here $t=0 > y=0$

$$\frac{dy}{dt} = e^{y} \cdot e^{-t} \sec(y)(1+t^2)$$

$$\frac{dy}{dt} = \frac{e^{y}}{e^{t}} \sec(y)(1+t^2)$$

$$dy/dt = e^{y}/e^{t} \; 1/\cos(y) \; (1+t^2)$$

$$e^{-y} \cos(y)\, dy = e^{-t}(1+t^2)\, dt$$

Now Integrating b/s

$$\int e^{-y}\cos(y)dy = \int e^{-t}(1+t^2)dt \rightarrow ①$$

$$\cos y \cdot \frac{e^{-y}}{-1} - \int -\sin y \cdot \frac{e^{-y}}{-1}dy = \int e^{-t}(1+t^2)dt$$

$$-e^{-y}\cos y - \left[-\sin y \cdot e^{-y} + \int \cos y \cdot e^{-y}dy\right] = \int e^{-t}(1+t^2)dt$$

$$-e^{-y}\cos y + e^{-y}\sin y - \int \cos y \cdot e^{-y}dy = \int e^{-t}(1+t^2)dt$$

$$-e^{-y}\cos y + e^{-y}\sin y - \int \cos y \cdot e^{-y}dy = \int e^{-t}(1+t^2)dt$$

we have

$$e^{-y}(\sin y - \cos y) = 2\int e^{-t}(1+t^2)dt$$

$$e^{-y}(\sin y - \cos y) = 2\left[(1+t^2)\frac{e^{-t}}{-1} \int 2t \cdot e^{-t}dt\right]$$

$$e^{-y}(\sin y - \cos y) = 2\left[-e^{-t}(1+t^2) + 2\left(\frac{t \cdot e^{-t}}{-1} + \int e^{-t}dt\right)\right]$$

$$e^{-y}(\sin y - \cos y) = 2\left[-e^{-t}(1+t^2) + 2(-te^{-t} - e^{-t}) \right] + c$$

$$e^{-y}(\sin y - \cos y) = 2\left[-e^{-t} - e^{-t}.t^2 - 2t \cdot e^{-t} - 2e^{-t} \right] + c$$

$$e^{-y}(\sin y - \cos y) = 2\left[-3e^{-t} - t^2 e^{-t} - 2t \cdot e^{-t} \right] + c$$

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -e^{-t}(t^2+2t+3) + c$$

Applying initial condition

$$y(0) = 0$$

$$\frac{1}{2}(-1) = -(3) + c$$

$$-\frac{1}{2} = -3 + c$$

$$\Rightarrow c = 3 - \frac{1}{2}$$

$$\Rightarrow \boxed{c = 5/2}$$

$$e^{-y}(\sin y - \cos y) = -2e^{-t}(t^2+2t+3) + 5/2$$

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Q No. 2

$(\sqrt{x+y}+\sqrt{x-y})dx-(\sqrt{x+y}-$

$\sqrt{x-y})dy=0$

$=(\sqrt{x+y}+\sqrt{x-y})dx-(\sqrt{(x+y)}]dy=0$

$=(\sqrt{x+y}+\sqrt{x-y})dx=\sqrt{x+y}-\sqrt{x+y})dy$

$\div$ing $\sqrt{x+y}$ & $\sqrt{x-y}$ on b/s.

$$\frac{\sqrt{x+y}}{\sqrt{x+y}-\sqrt{x-y}}+\frac{\sqrt{x-y}}{\sqrt{x+y}\sqrt{x-y}}dx\ \frac{\sqrt{x+y}}{(\sqrt{x+y})(\sqrt{x-y})}$$

$$-\frac{\sqrt{x-y}}{(\sqrt{x+y})(x-y)}dy$$

$$=\frac{1}{\sqrt{x-y}}+\frac{1}{\sqrt{x+y}}dx=\frac{1}{\sqrt{(x-y)}}-\frac{1}{\sqrt{x+y}}dy$$

Taking square.

$$\frac{1}{x-y}+\frac{1}{x+y}dx=\frac{1}{x-y}-\frac{1}{x+y}dy$$

$$\int\frac{1}{xy}dx+\int\frac{1}{x+y}dx-\int\frac{1}{xy}dy$$

$$-\int\frac{1}{x+y}dy$$

$-\ln|x-y| + \ln|x+y| + c = -\ln|x-y|$

$-\ln|x+y| + c$

$= \ln|(x-y)(x+y)| + \ln|(x-y)(x+y)| = c$

$= \ln|(x-y)(x+y)| + (x-y)(x+y) = \ln c$

$$\boxed{c = |(x-y)(x+y)| + (x-y)(x+y)}$$

Q No. 3

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

Sol:- $\frac{d^4y}{dx} + \frac{d^2y}{dx} = 3x^2 + 4\sin x - 2\cos x$

Since; $y = y_c + y_p \rightarrow ①$

y_c is associated with homo-genous part that is $m^4 + m^2 = 0 = (A.E)$

$\Rightarrow m^2(m^2+1) = 0$

$\Rightarrow m_1 = 0,\ m_2 = 0,\ m_3 = 2,\ m_4 = -1$

Thus $\boxed{y_c = C_1 + C_2x + C_3\cos x + C_4\sin x}$

Also to find y_p we use undetermined co-efficients approach.

$$y_p = (Ax^2 + Bx + C) + Dx\cos x + Ex\sin x \rightarrow ①$$

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$y(P) = 2Ax + B + Dx(-\sin x) + D\cos x +$

$Ex\cos x + E\sin x$ (i)

$y(P) = 2A - Dx\cos x - D\sin x - D\sin x$

$- Ex\sin x + E\cos x + E\cos x$

$y(P) = Dx\sin x + \cancel{D\cos x} - \cancel{D\cos x}$

$- D\cos x - Ex\cos x + \cancel{E\sin x} - \cancel{E\sin x}$

$- E\sin x$

$y(P) = Dx\sin x - D\cos x - Ex\cos x -$

$E\sin x$

$y(P) = Dx\cos x + \sin x + D\sin x + Ex\sin x$

$- E\cos x - E\cos x$

Putting values in original D.E and comparing co-efficients.

$\Rightarrow Dx\cos x + D\sin x + Ex\sin x$

$- E\cos x - E\cos x - 2A - Dx\cos x -$

$2D\sin x - Ex\sin x + 2E\cos x$

$= 3x^2 + 4\sin x - 2\cos x$

$2A$

$A = \frac{1}{2}$

$B = C = D = E = 0$

So; $y_P = \frac{1}{2} x^2$

$$\boxed{y = y_c + y_P}$$

$$\boxed{y = C_1 + C_2 x + C_3 \cos x + C_4 \sin x + \frac{1}{2} x^2}$$