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Section

B

Submitted to

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Subject

Intro to structural
Dynamics & earthquakes

Q#1

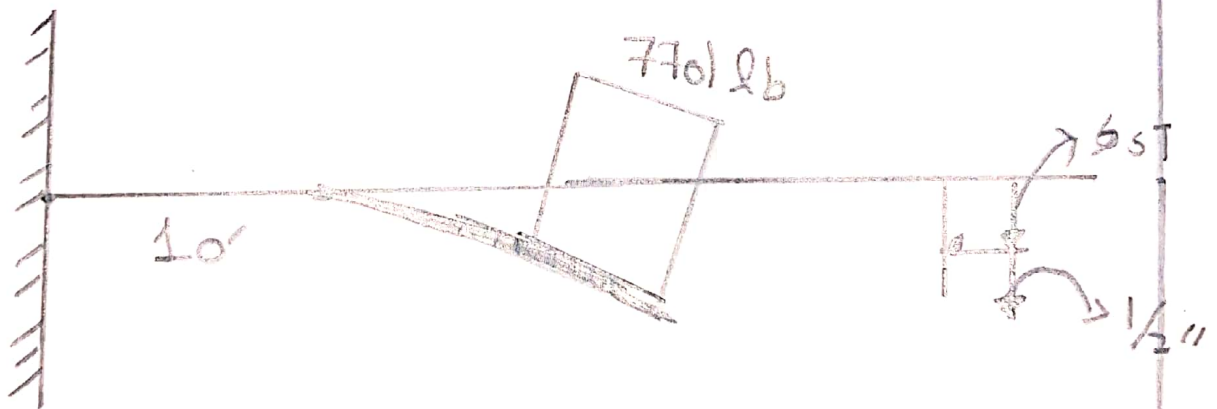
Given Data:

$$E = 29,000 \text{ ksi}$$

$$I = 150 \text{ in}^4$$

$\delta_r =$ deflection due to 770 lb

Static load beam is pulled $\frac{1}{2}$ " down words.



Required Data:

- Natural time period of system.
- Develop equation and solve the of motion.
- Draw graph to show

the variation of equivalent static force with time.

Solution:

General equation of motion for SDOF system is,

$$Ku + cu + mu = p(t)$$

Since system is undamped ($c=0$) under going for vibration $p(t)=0$

Hence general EOM become

$$Ku + mu = 0 \rightarrow \textcircled{A}$$

$$K = \frac{3EI}{L^3} = \frac{3 \times 29000 \text{ K/m}^2 \times 150}{(10 \times 12 \text{ in})^3}$$

$$K = 7.55 \text{ K/m.}$$

→ In order to estimate chances of mistake during calculation, it is more appropriate to use fundamental unit like lb, ft, sec or kg, m, sec.

$$K = 7.55 \text{ k/in}$$

$$K = 90.625 \text{ lb/ft.}$$

$$w = \frac{m}{g} = \frac{7701}{32.2} = \boxed{239.16 \text{ slugs}} \quad (3)$$

$$w_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{90625}{239.16}}$$

$$= \boxed{19.46 \text{ rad/sec}}$$

$$T_n = \frac{2\pi}{w_n} = \frac{2(3.14)}{19.46}$$

$$= \boxed{0.322 \text{ sec}}$$

put the value of m and k in equation 1.

$$= 90625u + 239.16\ddot{u} = 0$$

where k is in lb/ft and m is in $\text{lbsec}^2/\text{ft}^2 = \text{slugs}$.

general solution to EOM for undamped free vibrations is.

$$u(t) = u(0) \cos(\omega_n t) + \frac{u'(0)}{\omega_n \sin(\omega_n t)}$$

$$u(0) = \frac{1}{2}'' = \frac{1}{24} \text{ ft} \text{ and } u'(0) = 0$$

$$u(t) = \frac{1}{24} \times \cos(19.46t) + 0$$

$$= \frac{1}{24} \times \cos(19.46t)$$

Equivalent static force at any time "t" is

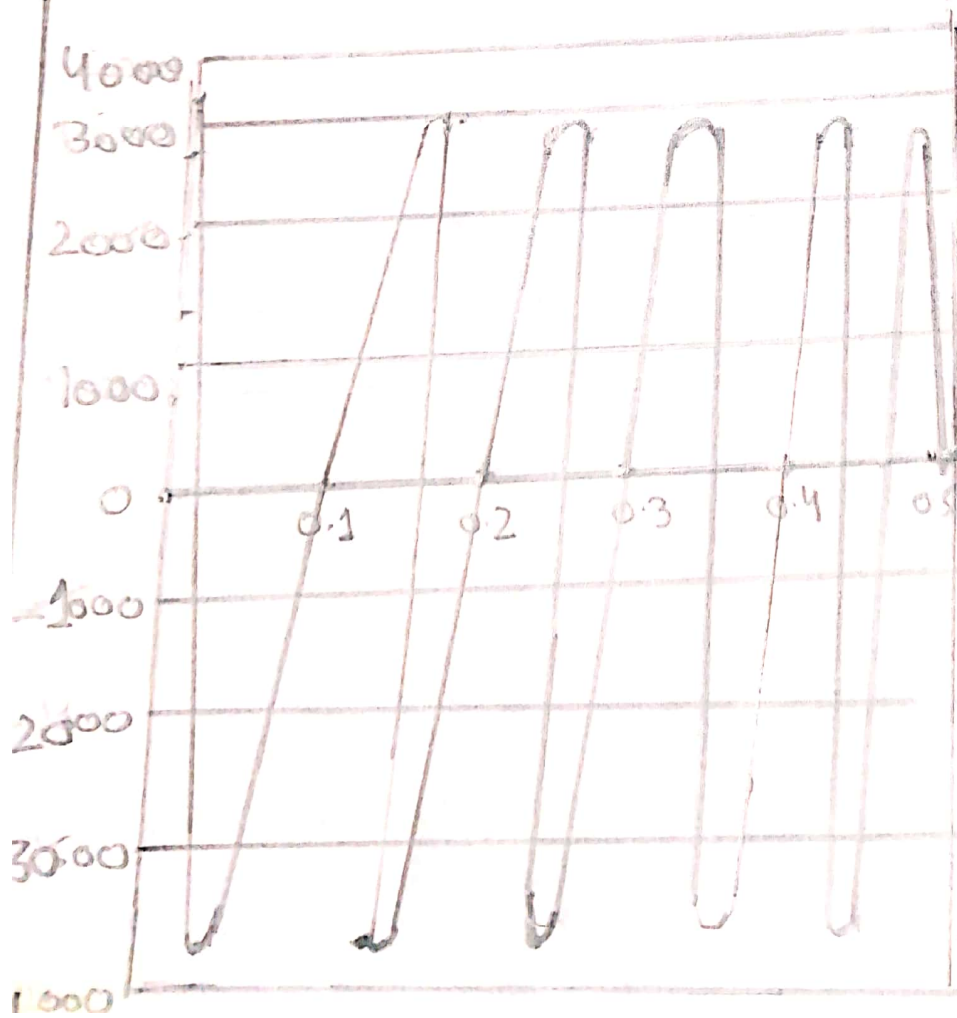
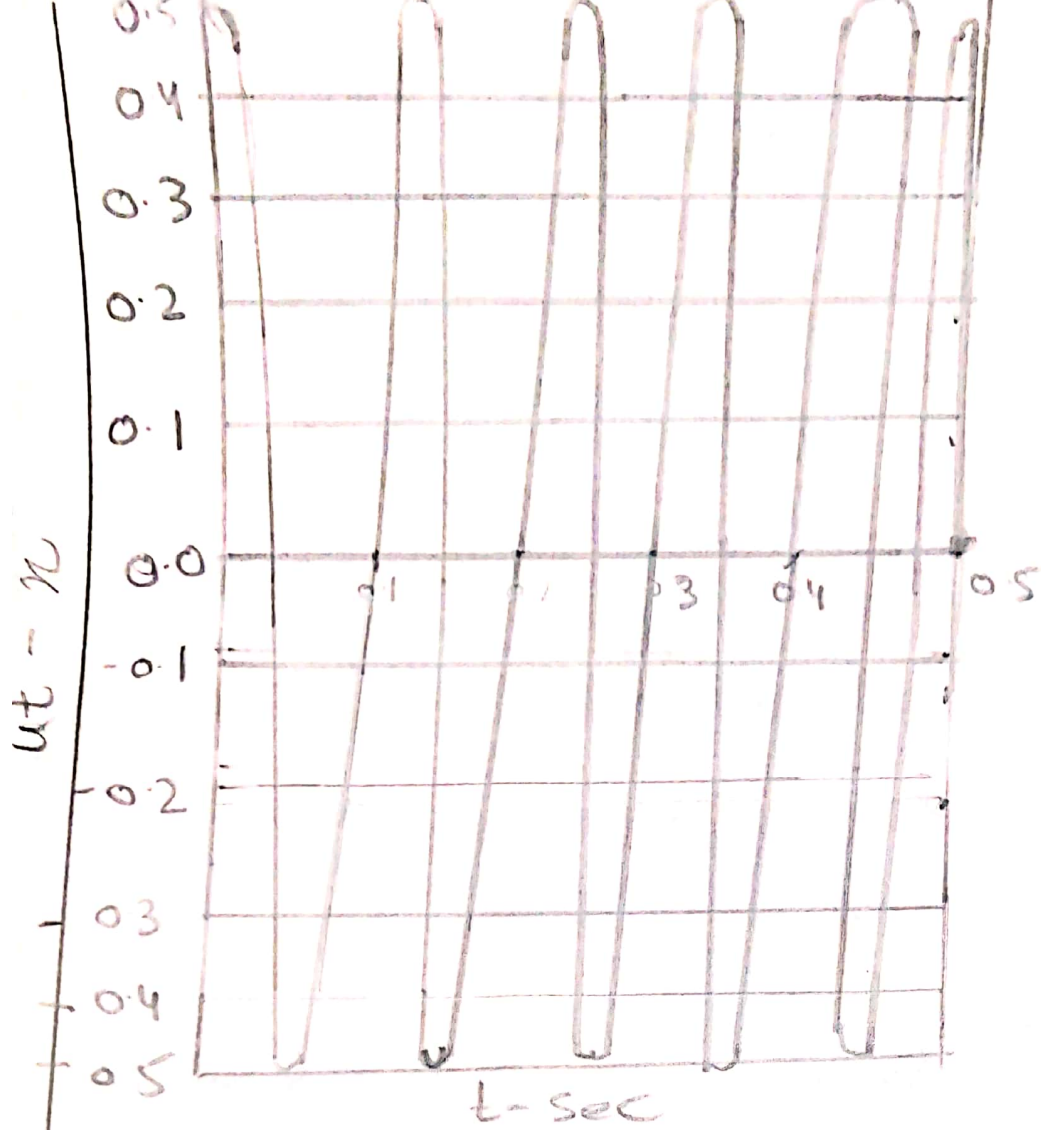
$$\begin{aligned}
 F_s(t) &= K \cdot u(t) \\
 &= \frac{90625 \times \cos(19.46t)}{24} \\
 &= \boxed{3776 \cos(19.46t)}
 \end{aligned}$$

Amplitude of dynamics displacement u_0 for undamped free vibration is

$$\begin{aligned}
 u_0 &= \sqrt{\left[(x(0))^2 + \left(\frac{\dot{x}(0)}{\omega_n} \right)^2 \right]} \\
 &= \sqrt{\left[\left(\frac{1}{24} \right)^2 + 0 \right]} \\
 &= \sqrt{\left(\frac{1}{24} \right)^2} \\
 &= \frac{1}{24} \text{ ft}
 \end{aligned}$$

Amplitude of equivalent static force

$$\begin{aligned}
 F_{s0} &= K u_0 = 90625 \times \frac{1}{24} \\
 &= 3776 \text{ lb}
 \end{aligned}$$



Q#2

Given Data:
 δ (damping ratio) of reinforced concrete with consideration cracking
= 3-5%
= 3%

Data of beam from Question #1

Required:

- Develop and solve the equation showing variation in equivalent static force with time.
- Draw graph to show the variation of displacement with time and the variation of equivalent static force with time.

Solution:

EOM for damped free vibration is

$$Ku + Cu + mu'' = 0 \rightarrow \textcircled{A}$$

As ^{question} from Equation 1

$$k = 90625 \text{ lb/ft}, \quad m = 239.16 \text{ lb sec}^2/\text{ft}$$

$$\omega_n = 19.46 \text{ rad/sec}$$

$$c = \zeta \times 2m \omega_n$$

$$\begin{aligned} c &= 3\% \times 2(239.16)(19.46) \\ &= 0.03 \times 2(239.16)(19.46) \\ &= 279.24 \text{ lb. sec/ft} \end{aligned}$$

put the value in equation (A)

$$= 90625 + 239.16 + 24034279.24 = 0$$

Solution to the ~~low~~ EOM for damped free vibrations

$$u(t) = e^{-\zeta \omega_n t} \left[u(0) \cos(\omega_d t) + \frac{1}{\omega_d} \left[\dot{u}(0) + \zeta \omega_n u(0) \right] \sin(\omega_d t) \right]$$

$$\omega_d = 19.46$$

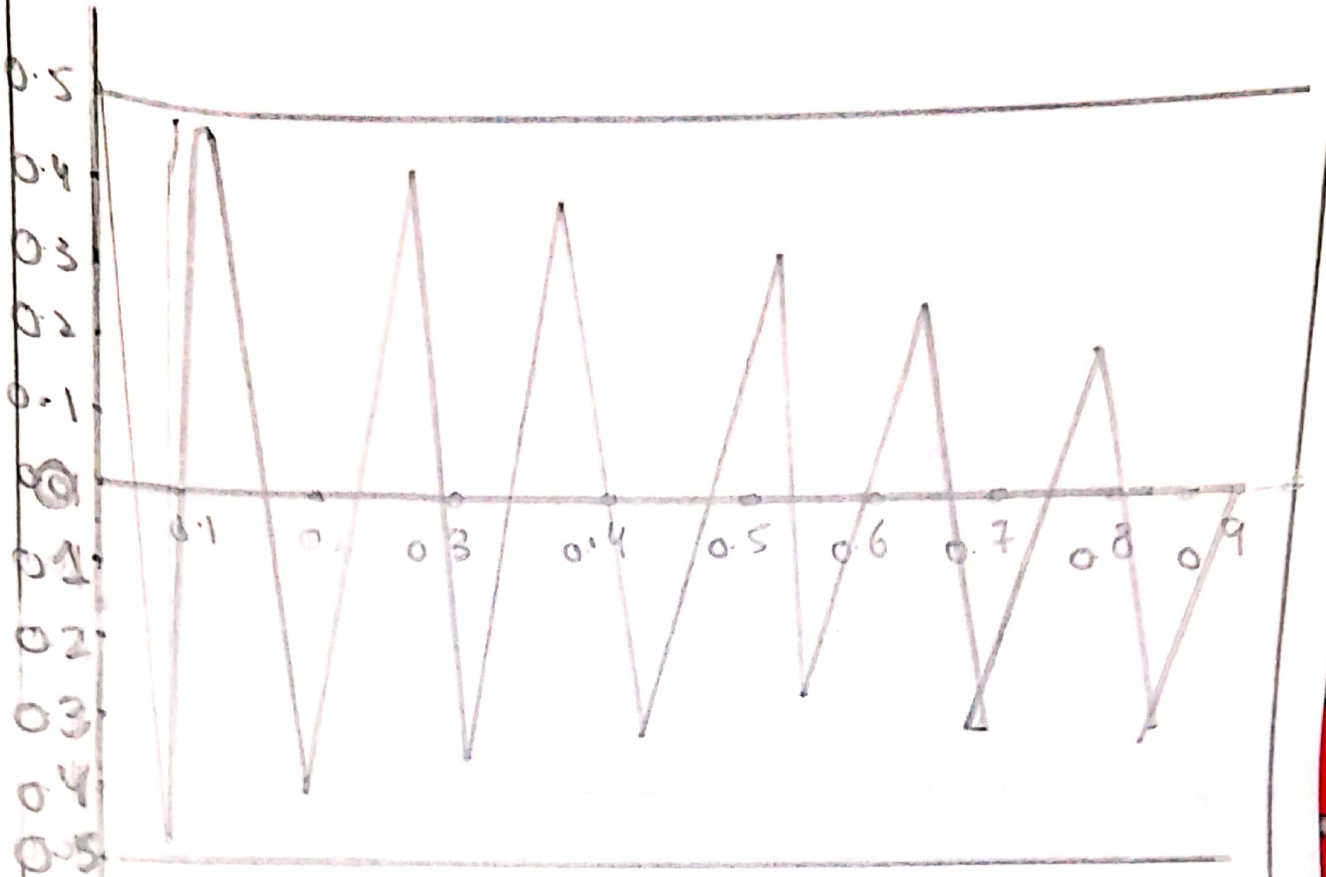
$$u(t) = e^{-0.583t} \left[0.041 x \cos(19.46t) + 0.00125 x \sin(19.46t) \right]$$

$$F_s(t) = k \cdot u(t) = 90625 x u(t).$$

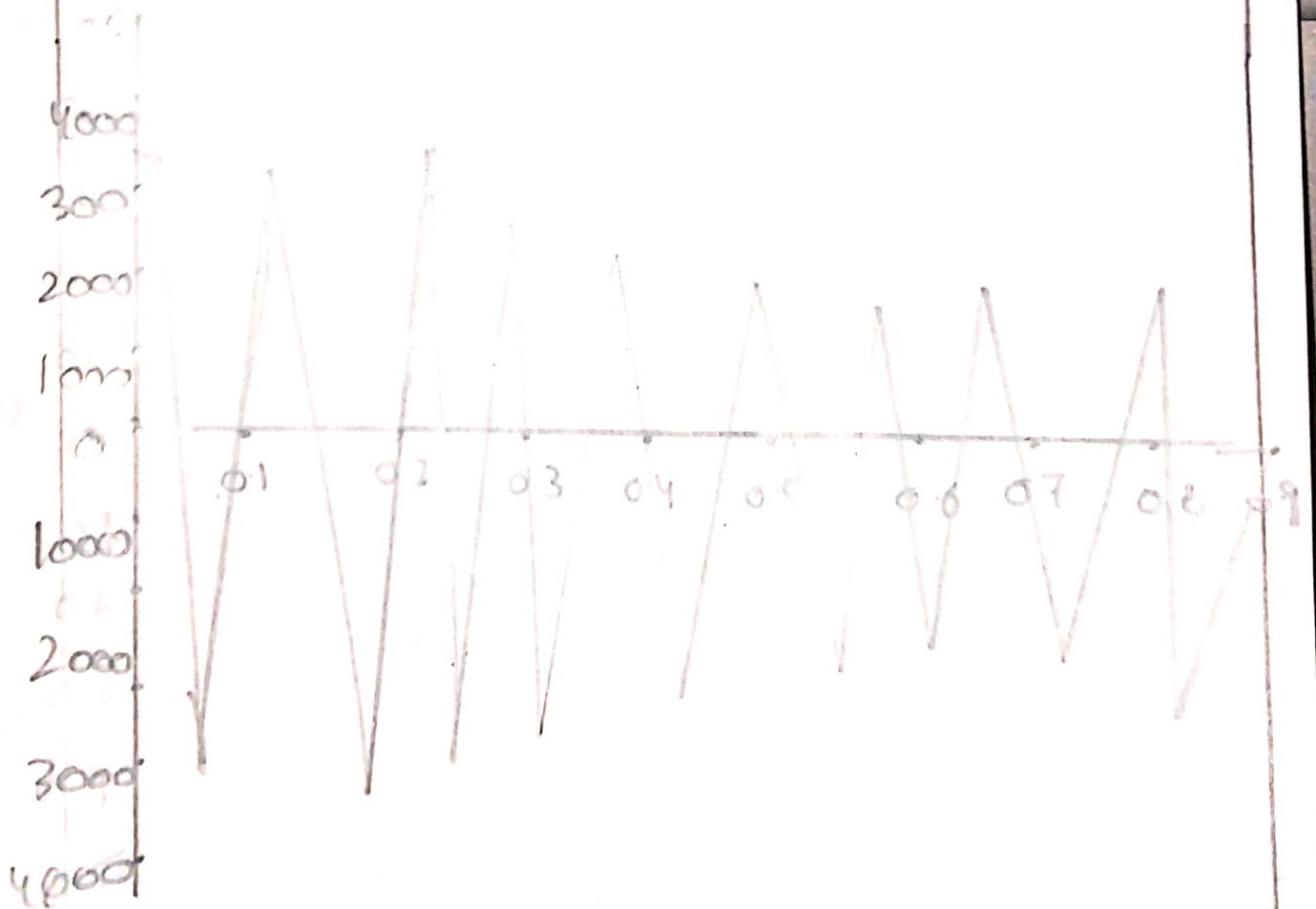
$$F_s(t) = e^{-0.583t} \left[(90625 \times 0.041) \cos(19.46t) + 90625 \times 0.00125 \sin(19.46t) \right]$$

$$F_s(t) = e^{-0.583t} \left(3715.62 \cdot \cos(19.46t) + 113.28 \right)$$

$u(t) = 1 - 2t$



$t - \omega(t)$



Q No # 3

Given data:

Force = 60 kips

Displacement of tank = $\frac{10}{1000} = \frac{770}{1000}$
 $= 7.70''$

Time taken to complete 7 cycles = 3.57

Amplitude of displacement = 2.286 cm
 $= 0.9''$

Required:

- ① Damping Ratio.
- ② Natural period of undamped vibration
- ③ stiffness of structure.
- ④ weight of tank.
- ⑤ damping coefficient.
- ⑥ The displacement amplitude to 0.5"
 Number of cycle to reduced.

Solution:

Displacement of tank = 7.70

After, 7 cycle.

$$J = 7, 4J + 1 = 48 = 0.9''$$

① Damping ratio:

$$J = \frac{1}{2\pi z} \ln \left[\frac{u_1}{u_1 + 1} \right]$$

~~$$7 = \frac{1}{2\pi z} \ln \left[\frac{7.70}{7.70 + 1} \right]$$~~

$$J = \frac{1}{2\pi z} \ln \left[\frac{7.70}{0.9} \right]$$

| |
|--------------|
| $z = 0.0488$ |
| $z = 4.88\%$ |

② Natural period of un damping vibration. $T_n = ?$

As 7, cycle of vibration are completed in 3.57 sec

→ Time Required to complete one cycle,

$$T_D = \frac{3.51}{7} = 0.51 \text{ sec}$$

$$T_D = 0.51 \text{ sec}$$

Now,

$$\omega_D = \omega_n \sqrt{1 - z^2}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n \sqrt{1 - z^2})}$$

$$T_D = \frac{T_n}{(1 - z^2)}$$

$$T_n = T_D \times \sqrt{1 - z^2}$$

$$T_n = 0.51 \times \sqrt{1 - (0.0488)^2}$$

$$T_n = 0.50944 = 0.51 \text{ sec}$$

$$T_n = 0.51 \text{ sec}$$

(c) stiffness of structure, $K = ?$

$$K = \frac{60 \times \cos 60^\circ}{7.70} = 3.896 \text{ K/in}$$

$$K = 3.9 \text{ K/in}$$

$$K = 46920 \text{ lb/ft}$$

(d) weight of tank $w = ?$

$$w_n = \sqrt{\frac{K}{M}} = \frac{K}{w/g}$$

$$\sqrt{\frac{K \cdot g}{w}}$$

taking square.

$$w_n = \sqrt{\frac{K \times g}{w}}$$

$$w_n^2 = \frac{K \times g}{w}$$

$$w = \frac{K \times g}{w_n^2}$$

also

$$w_n = \frac{2\pi}{T_n}$$

$$w = \frac{K \times g}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$= K \times g \frac{T_n^2}{4\pi^2}$$

$$= \left[\frac{46920}{\text{ft}} \times \frac{32.2 \text{ ft}}{\text{sec}^2} \right] \times \left[\frac{(0.5 \text{ sec})^2}{4\pi} \right]$$

$$= 9953.93$$

$$\boxed{= 9.95 \text{ K}}$$

⑥ Damping coefficient, $c = ?$

$$\zeta = \frac{c}{2m\omega_n}$$

$$c = \zeta \times 2m\omega_n$$

$$= 7 \times 2m \times \left(\frac{2\pi}{T_n} \right)$$

$$= \frac{0.0488 \times 4 \times \pi \times \frac{9953.9}{32.2}}$$

$$0.51$$

$$= 371.71 \text{ lb} \cdot \text{sec/ft}$$

F Number of cycle to reduce the displacement amplitude to $0.5''$ $J = ?$

$$J = \frac{1}{2\pi\zeta} \ln \left(\frac{u_1}{u_{J+1}} \right)$$

$$J = \frac{1}{2\pi \times 0.0488} \ln \left| \frac{7.70}{0.5} \right|$$

$$J = 8.91$$

$$\approx 9 \text{ cycles}$$

⑬

