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Subject	Differential equation
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Assignment	1

ODE (ordinary differential equation):-

An equation contains only ordinary derivates of one or more dependent variables of a single independent variable

For Example

$dy/dx + 5y = ex, (dx/dt) + (dy/dt) = 2x + y$

Application of ODE:-

1) Physical Application of ODE:-

i) Its velocity $(v) = \frac{dx}{dy}$

ii) Its acceleration $(a) = \frac{dv}{dt}$ or $\frac{d^2x}{dt^2}$ or $v\frac{dv}{dx}$

If however, the body be moving along a curve then

Its velocity $(v) = \frac{ds}{dt}$ or $v\frac{dv}{ds}$ or $\frac{d^2s}{dt^2}$

2) Newton's 2^{nd} Law:-

The rate of change in momentum encountered by a moving object is equal to the net force applied to it. In mathematical terms

$$F = \frac{d(mv)}{dt} \Rightarrow m\frac{dv}{dt} + v\frac{dm}{dt} \Rightarrow F = m\frac{dv}{dt}$$

$$\Rightarrow \boxed{F = ma}$$

3) Newton's Law of cooling:-

The rate of change of the temperature of an object is proportional to the difference between its own temperation and temperature of its surroundings

Therefore,

$\frac{d\theta}{dt} = EA(\theta - \theta r)$; E-A constant that depends upon the object, A - surface area, θ - A certain temperature, θr - Room/ambient temperature

4) Radioactive Half-Life:-

- A stochastic (random) process
- The rate of decay is dependant upon the number of molecules/atoms there are.
- Negative because the number is decreasing
- K is the constant of proportionality.

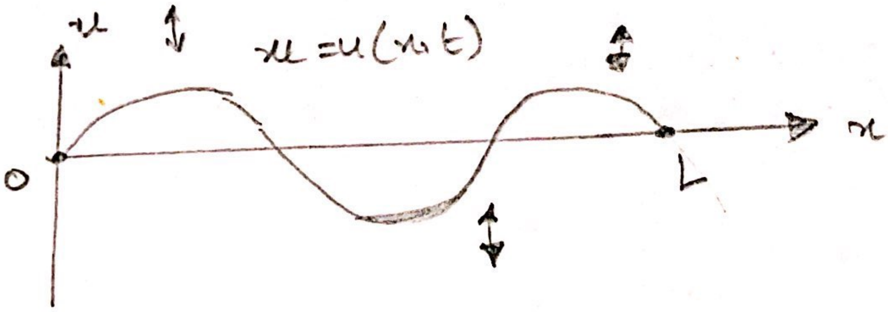
$$dN/dt = kN$$

Application of PDE (Partial differential equations):-

PDE's are used to mathematically formulate and thus aid the solution of, physical and other problems involving functions of several variables, such as the propagation of heat or sound, fluid flow, elasticity electrostatics, electrodynamics. etc.

1) Wave equation:-

The simplest situation to give rise to the one dimensional wave equation is the motion of a stretched string - specifically the transverse vibrations of a string such as the string of musical instrument



2) Laplace's equation:-

If you look back at the two-dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

It is clear that if the heat flow is steady i.e time independent then $\frac{\partial u}{\partial t} = 0$ so the temperature $u(x,y)$ is a solution of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

3) Heat conduction equation:-

Consider a long thin bar, or wire, of constant cross-section and of homogeneous material oriented along the x-axis

Imagine that the bar is thermally insulated laterally and is sufficient thin that heat flows (by conduction) only in the x-direction. Then the temperature u at any point in the bar depends only on the x-coordinate of the point and the time t., By applying the principle of conservation of energy it can be shown that $u(x,t)$ satisfies the PDE

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \qquad \begin{array}{c} 0 \le x \le L \\ t > 0 \end{array}$$

where k is a positive constant. In fact k, sometimes called the thermal diffusivity of the bar is given by

$$k = \frac{K}{s\rho}$$

where

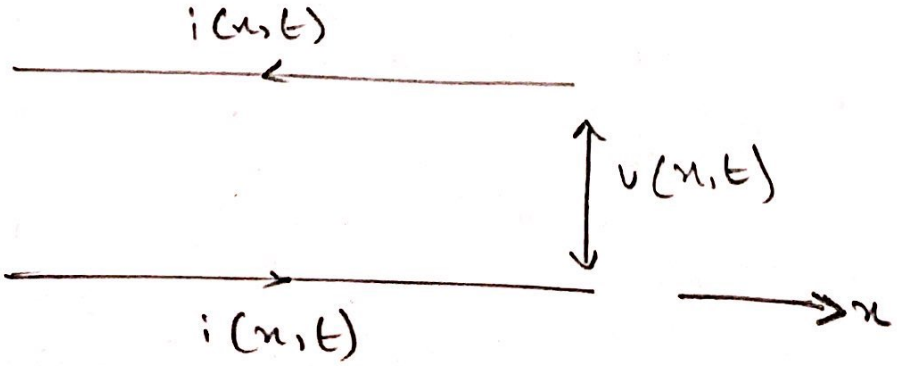
K = thermal conductivity of material of the bar

s = specific heat capacity of the material of the bar

ρ = density of the material of the bar.

4) Transmission line equations:-

In a long electrical cable or a telephone wire both the current and voltage depend upon position along the wire as well as the time



It is possible to show, using basic laws of electrical circuit theory, that the electric current $i(x,t)$
satisfies the PDE

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} + (RC + GL)\frac{\partial i}{\partial t} + RGi$$

where the constants R, L, C and G are, for unit length of cable, respectively the resistance, inductance, capacitance and leakage conductance. For a submarine cable G is negligible and frequencies are low so inductive effects can be neglected in that case

$$\frac{\partial^2 i}{\partial x^2} = RC\frac{\partial i}{\partial t}$$

which is called submarine equation.
For high frequency alternating currents again with negligible leakage

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2}$$

which is ~~eq~~ called high frequency line equation.