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Paper:- differential equation

Section:- 13 Civil.

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Que No - 01

Part (1).

$$w = \sin(x+ct) + \cos(2x+2ct)$$

$$\boxed{\frac{\partial^2 w}{\partial t^2} = \frac{c\,\partial^2 w}{\partial x^2}}$$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\frac{\partial^2 w}{\partial t} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2 \rightarrow (i)$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct) + 2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= [-\sin(x+ct) - 4\cos(x+2ct)]$$

$$\frac{\partial^2 w}{\partial t^2} = tc^2[-\sin(x+4) - 4(\cos(2x+2ct)]$$

$$c^2 \cdot \frac{\partial^2 w}{\partial x^2}$$

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$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$(1) \Rightarrow$

$$-c^2\sin(x+ct) - 4c^2\cos(2x+2ct) = c^2[-\sin(x+2ct) - 4\cos(mnt)$$

$$-\sin(\cancel{x}+ct) - 4c^2\cos(\cancel{2x}+2ct) = -c\cos\cancel{n}(x+ct) - 4c^2\cancel{\cos}(c4sct)$$

$0 = 0$ (Satisfied).

QNo#01 (Part)(ii)

$W = \tan(2x+ct)$

Now $\frac{\partial w}{\partial t} = c\sec^2(2x+ct)$

$= cg^2 \cdot \partial^2 \sec(2x+ct)\tan(2x+ct)$

Now

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 4\sec^2(2x+ct)\tan(2x+ct)$$

$(1) \Rightarrow 4c^2\sec^2(2x+ct)\cancel{\tan}(2x+ct) = 4c^2\sec^2(\cancel{2x}+ct)\tan(\cancel{2x}+ct)$

$0 = 0$ (Satisfied).

Q3: $y'' - 4y' + 13y = 8\sin 3x \rightarrow (1)$

Associated homogenous eq of eq (1) is

$y'' - 4y' + 13y = 0 \rightarrow (2)$

change equation (2) in auxilary equation.

put $y = m$ in eq (2)

$m^2 - 4m + 13 = 0$

use quadratic formula.

$a = 1,\ b = -4,\ c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2(a)}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$\Rightarrow \frac{4 \pm 6i}{2}$$

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$= \frac{4 \pm 6i}{2}$

$= \frac{4 \pm 3i}{2}$

$= 2 \pm 3i$

$m_1 = 2 + 3i$

$m_2 = 2 - 3i$

Let

$y_p = A\cos 3x + B\sin 3x \rightarrow (x)$

Diff: w.r.to "x"

$y_p' = -3A\sin 3x + 3B\cos 3x$

again Diff: w.r.to "x"

$y_p'' - 9A\cos 3x - 9B\sin 3x$

put in eq (1)

$\Rightarrow -9A\cos 3x - 12B\cos 3x + 13A\cos 3x$
$- 9B\sin 3x + 12A\sin 3x + 13B\sin 3x - 8\sin 3x$

$(-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x = 8\sin 3x$

Comparing coefficients

$\sin 3x \Rightarrow \quad 4B + 12A = 8 \rightarrow$ a

$\cos 3x \Rightarrow \quad 4A - 12B = 0 \Rightarrow 4A = 12B$

$\Rightarrow A = 3B \rightarrow$ b

put eq b in eq a

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$B = 1/5 \rightarrow$ c

put c in b

$A = 3/5 \rightarrow$ d

put c and d in eq x

$$y_p = \frac{B}{5} \cos x + \frac{1}{5} \sin 3x$$

The G. Sol is

$y = y_c + y_p$

$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5}\cos 3x$

$+ \frac{1}{5}\sin 3x \rightarrow (c)$

No we need to find the values of a and c for this put $x=0$ and $y=1$ in eq (c)

$1 = e^{(2)}(c_1 \cos_3(0) + c_2 \sin 3(0)) + \frac{3}{5}\cos 3(0)$
$+ \frac{1}{5}\sin 3(0)$

$1 = (C_1(1) + C_2(0)) + \frac{3}{5}(1) + \frac{1}{5}(0)$

$1 = C_1 + \frac{3}{5}$

$C_1 = 1 - \frac{3}{5}$

$C_1 = \frac{2}{5} \rightarrow (xx)$

Diff. (c) w-r-to "x"

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$$

$$-\frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x \rightarrow (D)$$

put $y' = 2,\ x = 0$ in (D)

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 2e^{2x}\cos 3x)$$

$$-\frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x$$

put $y' = 2 \quad x = 0$

$$2 = c_1(2e^{2(0)}\cos 3(0) + 3e^{2(0)}\cos 3(0)$$

$$-\frac{6}{5}\sin 3(0) + \frac{3}{5}\cos 3(0)$$

$$2 = c_1(2) + c_2\ 3 - 0, + \frac{3}{5}$$

$$2 = 2c_1 + 3c_2 + \frac{3}{5}$$

put $c_1 = \frac{2}{5}$

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$$2 = \frac{4}{5} + 3c_2 + \frac{3}{5}$$

$$2 = \frac{7}{5} + 3c_2$$

$$3c_2 = 2 - \frac{7}{5}$$

$$3c_2 = \frac{3}{5}$$

$$\boxed{c_2 = \frac{3}{15}} \longrightarrow (***)$$

put (**) and (***) in (C)

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right)$$

$$+ \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$$

$$\boxed{y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x}$$

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Given function is.

$$F(x) = \begin{cases} x ; & -\pi < x \le 0 \\ 2x ; & 0 \le x < \pi \end{cases}$$

We have to find find the fowerier Co-efficients, a_0, a_n & b_n

Now:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx.$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$a_0 = -\frac{\pi}{2} + \pi = \pi/2 \rightarrow (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x) \cos nx\,dx.$$

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$$= \frac{1}{\pi}\int_{-\pi}^{0} (f(x)\cos nx\, dx) + \frac{1}{\pi}\int_{0}^{\pi} (2x \cos nx)\, dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$\boxed{a_n = \begin{cases} \frac{-2}{\pi n^2} & ; \text{if } n \text{ is odd} \\ 0 & ; \text{if } n \text{ is even} \end{cases}} \longrightarrow (2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{-\pi\cos nx}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right]$$

$$= \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

So the required fourier series is:

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2x-1)x}{(2x-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}.$$