FINAL PAPER

NAME : Shah Sawar Khan

I.D : 7712

Section : A

Subject : Differential Equation

Submitted To : Ma'am Shumaila Mazhar.

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Q#01 Find the Fourier Series representation of $F(t) = 1+t, \; -\pi \le t \le \pi$

Sol:→ Here we use the formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \longrightarrow (1)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\,dt$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} (1+t)\,dt$$

$$a_0 = \frac{1}{2\pi}\left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi}\left(\pi-(-\pi)+\frac{\pi}{2}-\left(\frac{-\pi}{2}\right)^2\right)$$

$$a_0 = \frac{1}{2\pi}\left(2\pi+\frac{2\pi^2}{2}\right)$$

$$\boxed{a_0 = \frac{1}{2\pi}\left(2\pi+\pi^2\right)}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)(\cos nt)\,dt$$

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$$a_n = \frac{1}{\pi}\left((1+t)\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right)$$

$$a_n = \frac{1}{\pi}\left((1+t)\cancel{\frac{\sin nt}{n}}^{0} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\,dt$$

$$b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\int\sin nt\frac{d}{dt}(1+t)dt\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\frac{(1+t)(-\cos nt)}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt}{n}(1)\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\frac{-(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \left(\cancel{\frac{\sin nx}{n^2}}^{0}\Big|_{-\pi}^{\pi}\right)\right)$$

$$b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - (1+\pi)(\cos n(\pi))\right)$$

$$b_n = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi - \cancel{\cos n\pi} + \pi\cos n\pi\right)$$

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$$b_n = \frac{-1}{n\pi}\left(2\pi \cos n\pi\right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

So equation become

$$\boxed{f(x) = \frac{1}{2\pi}\left(2\pi + \pi^2\right) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin t}$$

Q#2 Calculate the characteristics equation the Eigen Values of the System where A is given by

$$\begin{matrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{matrix}$$

Sol:-

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen Values = ?

Step# 01:

we have

$(A - \lambda I)x = 0$ ∴ A is given matrix

Step# 02:

The equation is given by

$|A - \lambda I| = 0$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{bmatrix} = 0$$

Step# 03:

$$\lambda^3 - \left|\begin{matrix}\text{Sum of}\\ \text{Diagonal element}\end{matrix}\right| \lambda^2 + \left|\begin{matrix}\text{Sum of}\\ \text{diagonal}\\ \text{minor}\end{matrix}\right| \lambda - |A| = 0 \quad \text{— (B)}$$

Sum of diagonal element $= 1+1+2 = 4$

Sum of diagonal minor $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (+1)$

$= -6 + 2 + 1$

$= -3$

By putting values in eq (B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{— (C)}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$= 0$

Putting values in eq (C)

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$$\lambda(\lambda^2-4\lambda-3)=0$$

$$\lambda=0 \quad \text{or} \quad \lambda^2-4\lambda-3=0$$

Using Quadratic formula

$$\lambda=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad \therefore a=1,\ b=-4,\ c=-3$$

$$=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-3)}}{2(1)}$$

$$=\frac{4\pm\sqrt{16+12}}{2}$$

$$=\frac{4\pm\sqrt{28}}{2}$$

$$\lambda=\frac{4+\sqrt{28}}{2}, \quad \lambda=\frac{4-\sqrt{28}}{2}$$

So we have Eigen Values

$$\boxed{\lambda=\left(0,\ \frac{4+\sqrt{28}}{2},\ \frac{4-\sqrt{28}}{2}\right)}$$

is Required Answer

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Q#3 Solve the following System of linear equations

$$5x \quad + 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2z \quad = 1$$
$$x + y + z + m = 0$$

Sol:- writing in matrix form

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ m \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

The Augmented matrix "Ab" is

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_4 \times R_2$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \quad -1/5 \times R_3$$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad C_2 \times -5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad 5/4 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 120/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

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$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right] \quad 5\times R_2 \text{ and } 5\times R_3$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 5R_3 \; \&, \; 5R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 1/5\times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2\times 5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2\times 5,$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2/5 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3-R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = \left(3/4, 31/21, -11/21, 1/3\right)$$

So

$X = 3/4$

$Y = 31/21$

$Z = -11/21$

$m = 1/3$

Q#04 Verify that
$$u(x,t) = \sin(x+2t)$$
is solution of the one-dimensional wave equations

Sol:- Given that
$$u(x,t) = \sin(x+2t)$$
Differentiate w.r.t x partially

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\sin(x+2t)$$
$$\frac{\partial u}{\partial x} = \cos(x+2t)\frac{\partial}{\partial x}(x+2t)$$
$$\frac{\partial u}{\partial x} = \cos(x+2t)(1+0)$$
$$\frac{\partial u}{\partial x} = \cos(x+2t)$$
$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\cos(x+2t)$$

$$\frac{\partial^2 u}{dx^2} = -\sin(x+2t)\cdot\frac{\partial}{\partial x}(x+2t)$$
$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)(1+0)$$
$$\boxed{\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)}$$

and $u(x,t) = \sin(x+2t)$
differentiat w.r.t "t"

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t)\quad(0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t} = (2)-\sin(x+2t)\ (0+2)$$

$$\frac{\partial^2 u}{dt^2} = -4\sin(x+2t)$$

As we know that one-dimentional wave equation is

$$\frac{\partial^2 u}{dt^2} = c^2\frac{\partial^2 u}{dx^2}$$

$$-4\sin(x+2t) = c^2[-\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t)+c^2\sin(x+2t) = 0$$

For the orbitarary Constant $c = \pm 2$

$$-4\sin(x+2t)+(\pm 2)^2\sin(x+2t) = 0$$

$$-4\sin(x+2t)+4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for the orbitrary Constant
$C = 2.$