Sessional Assignment

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Mohr's Circles for 3-D Stress Analysis

The 3-D stresses, so called spatial stress problem, are usually given by the six stress components σ_x□, σ_y , σ_z , τ_{xy} , τ_{yz} , and τ_{zx} , (see Fig. 3) which consist in a three-by-three symmetric matrix (stress tensor):

$$\mathbf{T_3} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

(1)

What people usually are interested in more are the three prinicipal stresses σ_1 , σ_2 , and σ_3,which are eigenvalues of the three-by-three symmetric matrix of Eqn (16) ,and the three maximum shear stresses τ_{max1} , τ_{max2} , and τ_{max3} , which can be calculated from σ_1 , σ_2 , and σ_3.

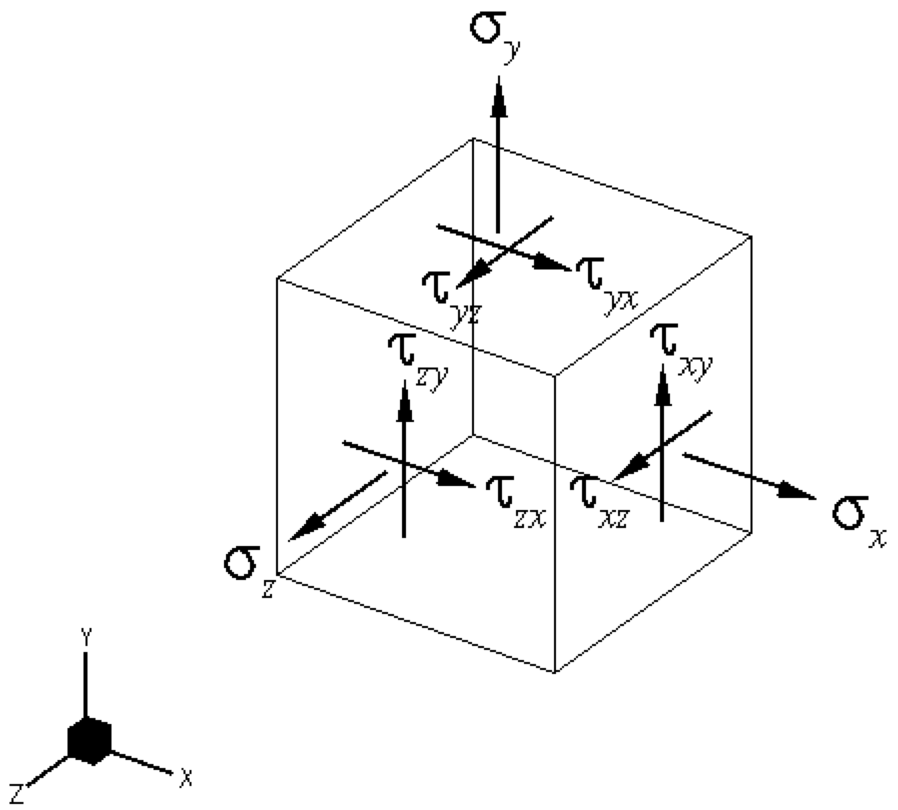


Fig. 3 3-D stress state represented by axes parallel to X-Y-Z

Imagine that there is a plane cut through the cube in Fig. 3 , and the unit normal vector **ν** of the cut plane has the direction cosines v_x , v_y , and v_z , that is

$$\boldsymbol{\nu} = (v_x , v_y , v_z) \tag{2}$$

then the normal stress on this plane can be represented by

$$\sigma_\nu = \sigma_x v^2_x + \sigma_y v^2_y + \sigma_z v^2_z + 2\,\tau_{xy} v_x v_y + 2\,\tau_{yz} v_y v_z + 2\,\tau_{xz} v_x v_z \tag{3}$$

There exist three sets of direction cosines, $\boldsymbol{\nu}_1$, $\boldsymbol{\nu}_2$, and $\boldsymbol{\nu}_3$ - the three principal axes, which make σ_ν achieve extreme values σ_1, σ_2, and σ_3 - the three principal stresses, and on the corresponding cut planes, the shear stresses vanish! The problem of finding the principal stresses and their associated axes is equivalent to finding the eigenvalues and eigenvectors of the following problem:

$$(\sigma \mathbf{I}_3 - \mathbf{T}_3)\boldsymbol{\nu} = \mathbf{0} \tag{4}$$

The three eigenvalues of Eqn (19) are the roots of the following characteristic polynomial equation:

$$\det(\sigma \mathbf{I}_3 - \mathbf{T}_3) = \sigma^3 - A\sigma^2 + B\sigma - C = 0 \tag{5}$$

where

$$A = \sigma_x + \sigma_y + \sigma_z \tag{6}$$

$$B = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \tau^2_{xy} - \tau^2_{yz} - \tau^2_{xz} \tag{7}$$

$$C = \sigma_x\sigma_y\sigma_z + 2\,\tau_{xy}\tau_{yz}\tau_{xz} - \sigma_x\tau^2_{yz} - \sigma_y\tau^2_{xz} - \sigma_z\tau^2_{xy} \tag{8}$$

In fact, the coefficients A, B, and C in Eqn (20) are invariants as long as the stress state is prescribed(see e.g. Ref. 2)Therefore, if the three roots of Eqn (20) are $\underline{\sigma}_1$, $\underline{\sigma}_2$, and $\underline{\sigma}_3$, one has the following equations:

$$\underline{\sigma}_1 + \underline{\sigma}_2 + \underline{\sigma}_3 = A \tag{9}$$

$$\underline{\sigma}_1\underline{\sigma}_2 + \underline{\sigma}_2\underline{\sigma}_3 + \underline{\sigma}_1\underline{\sigma}_3 = B \tag{10}$$

$$\underline{\sigma}_1\underline{\sigma}_2\underline{\sigma}_3 = C \tag{11}$$

Numerically, one can always find one of the three roots of Eqn (20) , e.g.$\underline{\sigma}_1$ using line search algorithm, e.g. bisection algorithm. Then combining Eqns (24)and (25), one obtains a simple quadratic equations and therefore obtains two other roots of Eqn (20), e.g.$\underline{\sigma}_2$ and $\underline{\sigma}_3$. To this end, one can re-order the three roots and obtains the three principal stresses, e.g.

$$\sigma_1 = \max(\underline{\sigma}_1 , \underline{\sigma}_2 , \underline{\sigma}_3)$$

(12)

$$\sigma_3 = \min(\underline{\sigma}_1, \underline{\sigma}_2, \underline{\sigma}_3)$$

(13)

$$\sigma_2 = (A - \sigma_1 - \sigma_2)$$

(14)

Now, substituting σ_1, σ_2, or σ_3 into Eqn (19), one can obtains the corresponding principal axes $\mathbf{v}_1$, $\mathbf{v}_2$, or $\mathbf{v}_3$, respectively.

Similar to Fig. 3, one can imagine a cube with their faces normal to $\mathbf{v}_1$, $\mathbf{v}_2$, or $\mathbf{v}_3$. For example, one can do so in Fig. 3 by replacing the axes **X**,**Y**, and **Z** with $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$, respectively, replacing the normal stresses σ_x, σ_y, and σ_z with the principal stresses σ_1, σ_2, and σ_3, respectively, and removing the shear stresses τ_{xy}, τ_{yz}, and τ_{zx}.

Now, pay attention the new cube with axes $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$. Let the cube be rotated about the axis $\mathbf{v}_3$, then the corresponding transformation of stress may be analyzed by means of Mohr's circle as if it were a transformation of plane stress. Indeed, the shear stresses excerted on the faces normal to the $\mathbf{v}_3$ axis remain equal to zero, and the normal stress σ_3 is perpendicular to the plane spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$ in which the transformation takes place and thus, does not affect this transformation. One may therefore use the circle of diameter *AB* to determine the normal and shear stresses exerted on the faces of the cube as it is rotated about the $\mathbf{v}_3$ axis (see Fig. 4). Similarly, the circles of diameter *BC* and *CA* may be used to determine the stresses on the cube as it is rotated about the $\mathbf{v}_1$ and $\mathbf{v}_2$ axes, respectively.

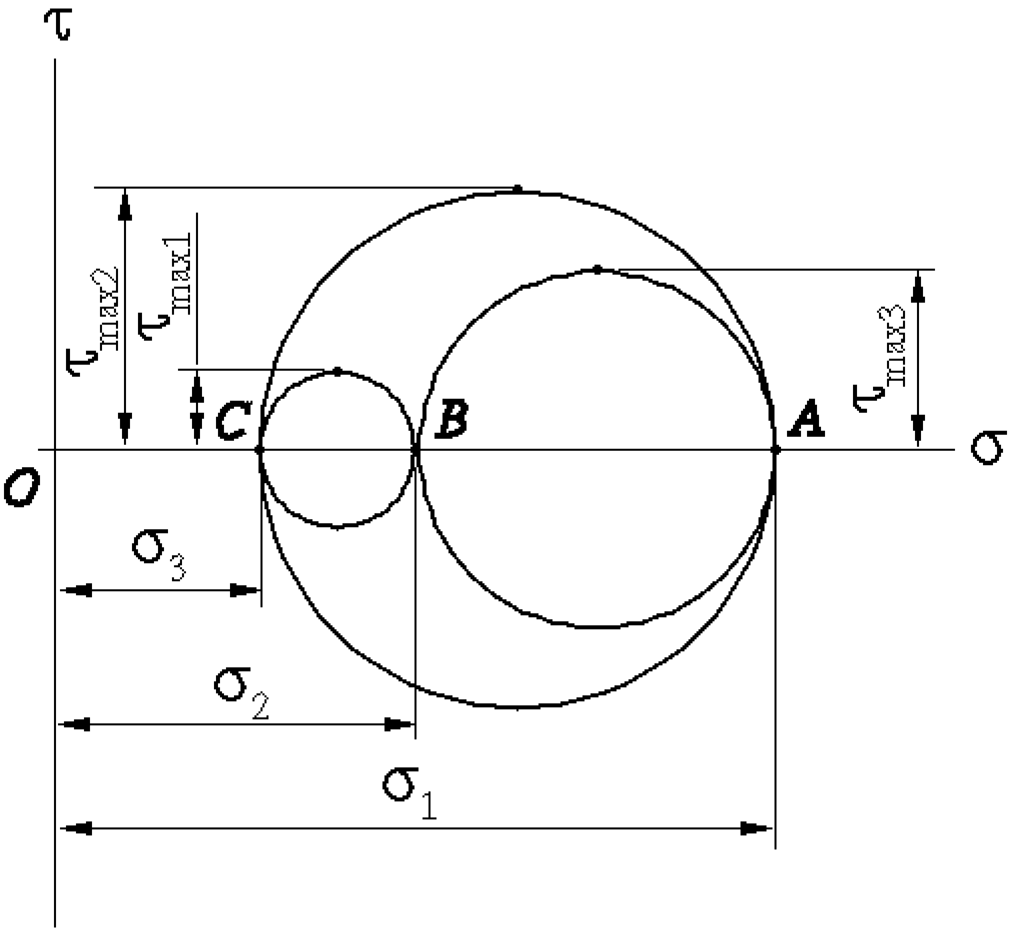


Fig. 4 Mohr's circles for space (3-D) stress

What if the rotations are about the axes rather than principal axes? It can be shown that any other transformation of axes would lead to stresses represented in Fig. 4 by a point located within the area which is bounded by the bigest circle with the other two circles removed!

Therefore, one can obtain the maxium/minimum normal and shear stresses from Mohr's circles for 3-D stress as shown in Fig. 4!

Note the notations above (which may be different from other references), one obtains that

$$\sigma_{max} = \sigma_1 \tag{15}$$

$$\sigma_{min} = \sigma_3 \tag{16}$$

$$\tau_{max} = (\sigma_1 - \sigma_3)/2 = \tau_{max2} \tag{17}$$

Note that in Fig. 4, τ_{max1}, τ_{max2}, and τ_{max3} are the maximum shear stresses obtained while the rotation is about $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ □, respectively.

DIMENSIONAL ANALYSIS

Dimensional analysis is a means of simplifying a physical problem by appealing to dimensional homogeneity to reduce the number of relevant variables. It is particularly useful for presenting and interpreting experimental data attacking problems not amenable to a direct theoretical solution checking equations establishing the relative importance of particular physical phenomena; • physical modelling. Example. The drag force F per unit length on a long smooth cylinder is a function of air speed U, density ρ, diameter D and viscosity μ. However, instead of having to draw hundreds of graphs portraying its variation with all combinations of these parameters, dimensional analysis tells us that the problem can be reduced to a single dimensionless relationship c f (Re) D = where cD is the drag coefficient and Re is the Reynolds number. In this instance dimensional analysis has reduced the number of relevant variables from 5 to 2 and the experimental data to a single graph of cD against Re.

Dimensional Formula of Stress

The dimensional formula of Stress is given by,

$$[M^1 L^{-1} T^{-2}]$$

Where,

- M = Mass
- L = Length
- T = Time

Derivation

Stress = Force × $[\text{Area}]^{-1}$ . . . . . (1)

The dimensional formula of area = $[M^0 L^2 T^0]$ . . . . (2)

Since, Force = M × a = [M] × $[M^0 L^1 T^{-2}]$

∴ The dimensional formula of force = $[M^1 L^1 T^{-2}]$ . . . . (3)

On substituting equation (2) and (3) in equation (1) we get,

Stress = Force × $[Area]^{-1}$

Or, Stress = $[M^1 L^1 T^{-2}] \times [M^0 L^2 T^0]^{-1} = [M^1 L^{-1} T^{-2}]$

Therefore, stress is dimensionally represented as **$[M^1 L^{-1} T^{-2}]$.**

Bending stresses are those that bend the beam because of beam self-load and external load acting on it.

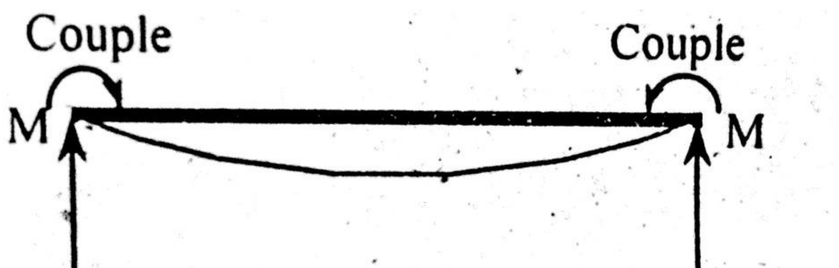
Bending stresses

Bending stresses are of two types;

1. Pure Bending
2. Simple Bending

Pure Bending:

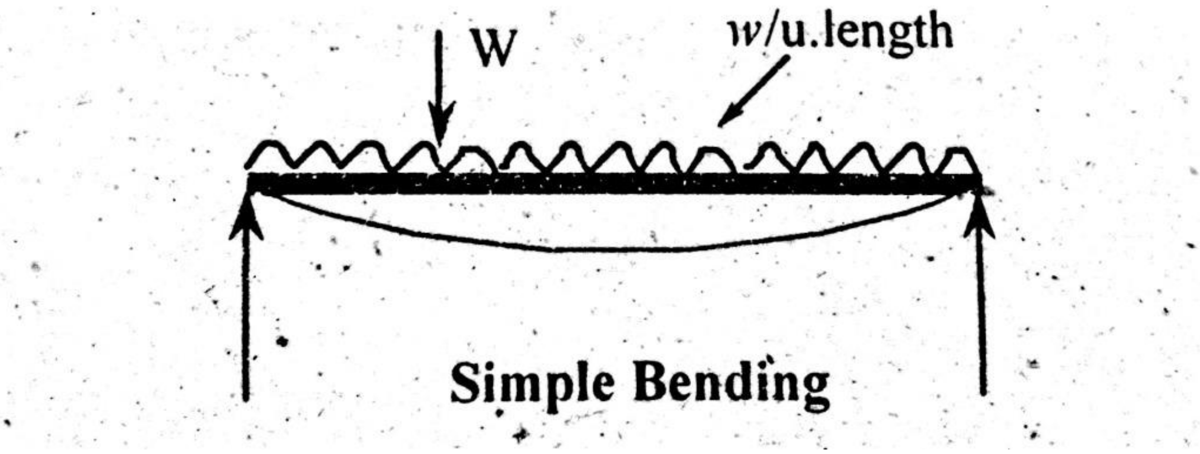
Bending will be called as pure bending when it occurs solely because of coupling on its end. In that case there is no chance of shear stress in the beam. But, the stress that will propagate in the beam as a result will be known as normal stress. Normal stress because it not causing any damages to beam. As shown below in the picture.



Pure Bending

Simple Bending:

Bending will be called as simple bending when it occurs because of beam self-load and external load. This type of bending is also known as ordinary bending and in this type of bending results both shear stress and normal stress in the beam. As shown below in the figure.

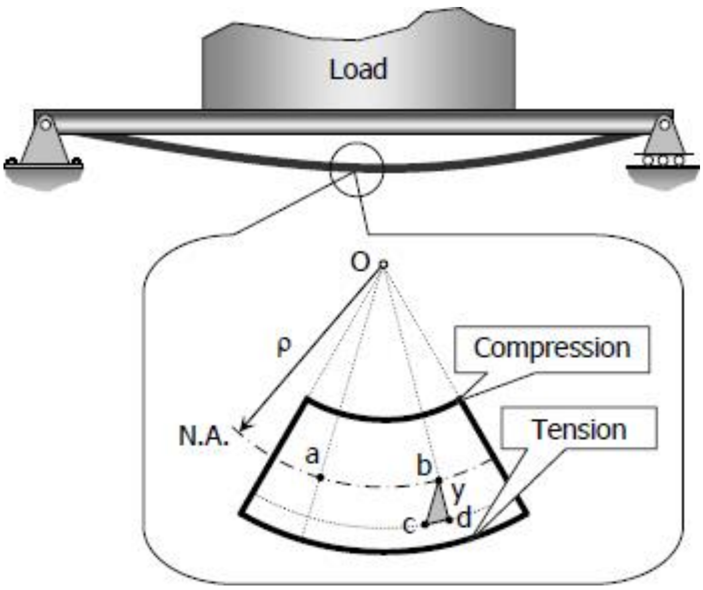


Assumption made in theory of pure bending

1. The material of the beam is homogeneous[1] and isotropic[2].
2. The value of Young's Modulus of Elasticity is same in tension and compression.
3. The transverse sections which were plane before bending, remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.
5. The radius of curvature is large as compared to the dimensions of the cross-section.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

Flexure Formula

Stresses caused by the bending moment are known as flexural or bending stresses. Consider a beam to be loaded as shown.



Consider a fiber at a distance yy from the neutral axis, because of the beam's curvature, as the effect of bending moment, the fiber is stretched by an amount of cdcd. Since the curvature of the beam is very small, bcdbcd and ObaOba are considered as similar triangles. The strain on this fiber is

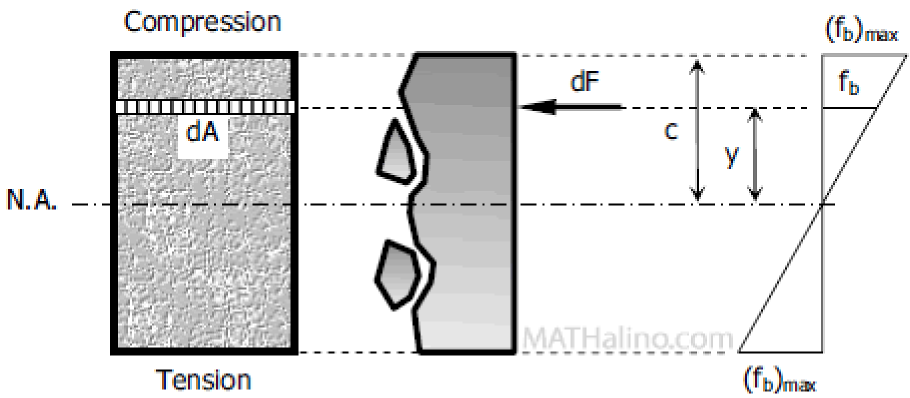
$$\varepsilon=cdab=y\rho\varepsilon=cdab=y\rho$$

By Hooke's law, $\varepsilon=\sigma/E\varepsilon=\sigma/E$, then

$$\sigma E=y\rho;\sigma=y\rho E\sigma E=y\rho;\sigma=y\rho E$$

which means that the stress is proportional to the distance yy from the neutral axis.

For this section, the notation fbfb will be used instead of σσ.



Considering a differential area dAdA at a distance yy from N.A., the force acting over the area is

dF=fbdA=yρEdA=EρydAdF=fbdA=yρEdA=EρydA

The resultant of all the elemental moment about N.A. must be equal to the bending moment on the section.

M=∫dM=∫ydF=∫y(EρydA)M=∫dM=∫ydF=∫y(EρydA)

M=Eρ∫y2dAM=Eρ∫y2dA

but ∫y2dA=I∫y2dA=I, then

M=EIρorρ=EIMM=EIρorρ=EIM

substituting ρ=Ey/fbρ=Ey/fb

Eyfb=EIMEyfb=EIM

then

fb=MyIfb=MyI

and

(fb)max=McI(fb)max=McI

The the bending stress due beam curvature is

fb=McI=EIρcIfb=McI=EIρcI

fb=Ecρfb=Ecρ

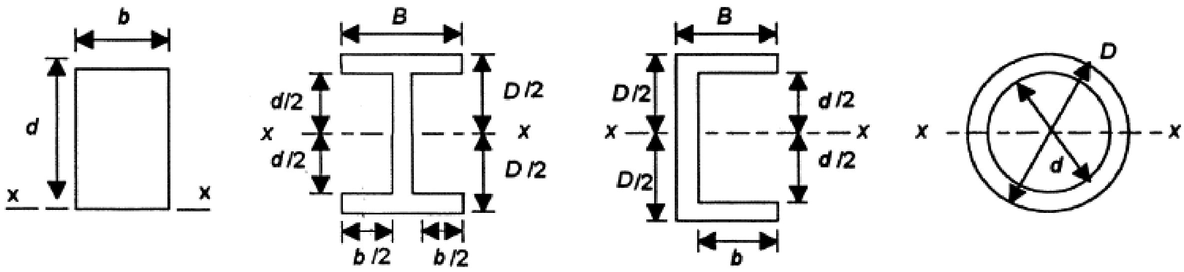
The beam curvature is:

k=1ρk=1ρ

where ρρ is the radius of curvature of the beam in mm (in), MM is the bending moment in N·mm (lb·in), fbfb is the flexural stress in MPa (psi), II is the centroidal moment of inertia in mm4 (in4), and cc is the distance from the neutral axis to the outermost fiber in mm (in).

Section modulus

Section modulus is a geometric property for a given cross-section used in the design of beams or flexural members. Other geometric properties used in design include area for tension and shear, radius of gyration for compression, and moment of inertia and polar moment of inertia for stiffness. Any relationship between these properties is highly dependent on the shape in question. Equations for the section moduli of common shapes are given below. There are two types of section moduli, the elastic section modulus and the plastic section modulus. The section moduli of different profiles can also be found as numerical values for common profiles in tables listing properties of such



Second moment of area I_{xx}	$\frac{bd^3}{3}$	$\frac{BD^3-bd^3}{12}$	$\frac{BD^3-bd^3}{12}$	$\frac{\pi(D^4-d^4)}{64}$
Section modulus Z	$\frac{bd^2}{3}$	$\frac{BD^3-bd^3}{6D}$	$\frac{BD^3-bd^3}{6D}$	$\frac{\pi(D^4-d^4)}{32D}$

Application of Bending Equation in any object.

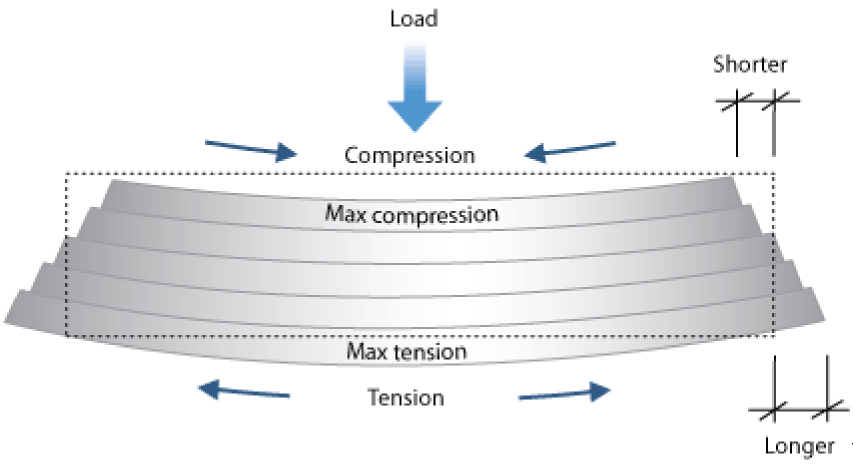
In applied mechanics, bending (also known as flexure) characterizes the behavior of a slender structural element subjected to an external load applied perpendicularly to a longitudinal axis of the element.

The structural element is assumed to be such that at least one of its dimensions is a small fraction, typically 1/10 or less, of the other two. When the length is considerably longer than the width and the thickness, the element is called a beam. For example, a closet rod sagging under the weight of clothes on clothes hangers is an example of a beam experiencing bending. On the other hand, a shell is a structure of any geometric form where the length and the width are of the same order of magnitude but the thickness of the structure (known as the 'wall') is considerably smaller. A large diameter, but thin-walled, short tube supported at its ends and loaded laterally is an example of a shell experiencing bending.

In the absence of a qualifier, the term *bending* is ambiguous because bending can occur locally in all objects. Therefore, to make the usage of the term more precise, engineers refer to a specific object such as; the *bending of rods*, the *bending of beams*,[1] the *bending of plates*, the *bending of shells* and so on

Internal moment of resistance

When a beam bends under load, the horizontal fibres will change in length. The top fibres will become shorter and the bottom fibres will become longer. The most extreme top fibre will be under the greatest amount of **compression** while the most extreme bottom fibre will be under the greatest amount of **tension**.



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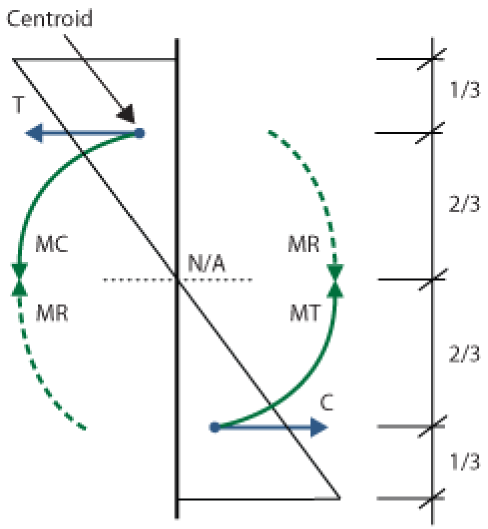
Application of Bending Equation in any object.

An engineer determines the centroids of the triangular shapes of the stress diagram. This provides the value of the total compressive and tensile forces acting in the beam. These centroids are a proportional distance apart which is referred to as the moment arm**.**

The moment arm provides the value for moments that the beam must resist if it is to remain structurally sound. In technical terms it is referred to as the



The tensile and compressive stresses result in a turning effect about the neutral axis. These are called moment $\mathbf{M}_T$ and $\mathbf{M}_C$ respectively. The chosen beam must be able to resist these moments with $\mathbf{M}_R$ (internal moment of resistance) if it is to remain in equilibrium.

ECCENTRICALLY LOADED COLUMNS: AXIAL LOAD AND BENDING.

Members that are axially, i.e., concentrically, compressed occur rarely, if ever, in buildings and other structures. Components such as columns and arches chiefly carry loads in compression, but simultaneous bending is almost always present. Bending moments are caused by continuity, i.e., by the fact that building columns are parts of monolithic frames in which the support moments of the girders are partly resisted by the abutting columns, by transverse loads such as wind forces, by loads carried eccentrically on column brackets, or in arches when the arch axis does not coincide with the pressure line. Even when design calculations show a member to be loaded purely axially, inevitable imperfections of construction will introduce eccentricities and consequent bending in the member as built. For this reason members that must be designed for simultaneous compression and bending are very frequent in almost all types of concrete structures. When a member is subjected to combined axial compression and moment, such as in the figure (a), it is usually convenient to replace the axial load and moment with an equal load applied at eccentricity, as in figure (b). The two loadings are statically equivalent. All columns may then be classified in terms of the equivalent eccentricity. Those having relatively small are generally characterized by 14 compression over the entire concrete section, and if overloaded, will fail by crushing of the concrete accompanied by yielding of the steel in compression on the more heavily loaded side. Columns with large eccentricity are subject to tension over at least a part of the section, and if overloaded, may fail due to tensile yielding of the steel on the side farthest from the load. For columns, load stages below the ultimate are generally not important. Cracking of concrete, even for columns with large eccentricity, is usually not a serious problem, and lateral deflections at service load levels are seldom, if ever, a factor. Design of columns is therefore based on the factored load, which must not exceed the design strength, as usual, i.e. The design limitations for columns, according to the ACI Code, Section 10.2, are as follows: 1. Strains in concrete and steel are proportional to the distance from the neutral axis. 2. Equilibrium of forces and strain compatibility must be satisfied. 3. The maximum usable compressive strain in concrete is 0.003. 4. Strength of concrete in tension can be neglected. 5. The stress in the steel is . 6. The concrete stress block may be taken as a rectangular shape with concrete stress of that extends from the extreme compressive fibers a distance , where is the distance to the neutral axis and where as defined in ACI 10.2.7.3 equal: 15 The eccentricity, , represents the distance from the plastic centroid of the section to the point of application of the load. The plastic centroid is obtained by determining the location of the resultant force produced by the steel and the concrete, assuming that both are stressed in compression to and , respectively. For symmetrical sections, the plastic centroid coincides with the centroid of the section

The theorem of least work derives from what is known as Castigliano's second theorem. So, let's first state the two theorems of Carlo Alberto Castigliano (1847-1884) who was an Italian railroad engineer. In 1879, Castigliano published two theorems. Castigliano's first theorem The first partial derivative of the total internal energy (strain energy) in a structure with respect to any particular deflection component at a point is equal to the force applied at that point and in the direction corresponding to that deflection component. This first theorem is applicable to linearly or nonlinearly elastic structures in which the temperature is constant and the supports are unyielding. Castigliano's second theorem The first partial derivative of the total internal energy in a structure with respect to the force applied at any point is equal to the deflection at the point of application of that force in the direction of its line of action. The second theorem of Castigliano is applicable to linearly elastic (Hookean material) structures with constant temperature and unyielding supports. Note that in the above statements, force may mean point force or couple (moment) and displacement may mean translation or angular rotation. Proofs of Castigliano's theorems are given at the end of this document. Without further due, here is the theorem of least work, a.k.a. Castigliano's theorem of least work: The redundant reaction components of a statically indeterminate structure are such that they make the internal work (strain energy) a minimum. Please read the above statement again. It is a succinct statement of Nature's tendency to conserve energy. (Or it could be interpreted as Nature prefers to be lazy1 .) We shall explain the proof of the theorem of least work and its application first by the use of a simple example shown below. A B P1 C P2 VA VB VC = A B P1 C P2 VA VB VC The beam shown on the left is statically indeterminate to the first degree. It is obvious that the simple determinate beam shown on the right is equivalent to the original beam on the left with a geometric condition (compatibility condition). That condition with which VB can be determined is that the deflection at B of the equivalent beam should be zero. This deflection, by Castigliano's second theorem, is B B U V ∂ Δ = ∂ . But we know that support B has zero vertical deflection. 1 It is said that it takes 43 muscles to frown and 17 muscles to smile (hence smiling ϑ is easier than frowning Λ), but none to do nothing. 2 Hence, the condition for determining VB becomes 0 B U V ∂ = ∂ , or VB is such as to make the total internal work a minimum. Note that when the first derivative of a function with respect to a variable and at a certain value of the variable is equal to zero, the function may be either a maximum or a minimum. Appealing to our sense of physics, we can eliminate the possibility that the total work can be a maximum. Hence the result: when Nature has its free choice, it will always tend to conserve energy. To give an example with two redundants (i.e. statically indeterminate to second degree), we can consider the following system. Choosing moment at A and vertical force at B as the redundants, we can obtain the equivalent system on the right. A P1 P2 VA VC MA B VB = The conditions of geometry together with Castigliano's second theorem state that 0 A A U M θ ∂ = = ∂ and 0 B B U V ∂ Δ= = ∂ which simply means that the redundants M A and VB are such that they minimize the total internal strain energy U.