Title

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Q1

Ans i) $W = \sin(x+ct) + \cos(2x + 2ct)$

Given
$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

Now:
$$\partial w/\partial t = \frac{\partial}{\partial t}\left[\sin(x+ct) + \cos(2x+2ct)\right]$$
$$= \partial/\partial t\,(\sin(x+ct)) + \frac{\partial}{\partial t}(\cos(2x+2ct))$$
$$\frac{\partial w}{\partial t} = c\cos(x+ct) - 2c\sin(2x+2ct)$$

Now
$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left[c\cos(x+ct) - 2c\sin(2x+2ct)\right]$$
$$\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)$$

Now $\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}\left[\sin(x+ct) - 2\sin(2x+2ct)\right]$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left[\cos(x+ct) - 2\sin(2x+2ct)\right]$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

1) $-c^2 \sin(x+ct) - 4c^2 \cos(2x+2ct) = c^2[-\sin(x+ct) - 4(\cos 2x+2ct)]$

$$-c^2 \cancel{\sin}(x+ct) - 4c^2 \cancel{\cos}(2x+2ct) = -\cos(\cancel{x+ct}) - 4c^2 \cancel{\cos}(2x+2ct)$$

$$0 = 0 \quad \text{(satisfied)}$$

ii) $W = \tan(2x+ct)$

Now $\frac{\partial W}{\partial t} = c\sec^2(2x+ct)$

& $\frac{\partial^2 W}{\partial t^2} = \frac{\partial}{\partial t}\left(c \sec^2(2x+xt)\right)$

$$= c_2^2 \cdot 2\sec^2(2x+ct)\tan(2x+ct)$$

Now $\frac{\partial W}{\partial x} = 2\sec^2(2x+ct)$

$$\frac{\partial^2 W}{\partial x^2} = 4\sec^2(2x+ct)\tan(2x+ct)$$

1) $\Rightarrow 4c^2 \cancel{\sec^2}(2x+ct)\tan(2x+ct)$

$$= 4c^2 \cancel{\sec^2}(2x+ct)\tan(2x+ct)$$

$$0 = 0 \quad \text{(satisfied)}$$

Q2
Answer Given function is

$$f(x) \begin{cases} x & : -\pi < x \leq 0 \\ 2x & : 0 \leq x \leq \pi \end{cases}$$

we have to find the fourieor co-efficents a_0, a_n & b_n

Now

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dz = \tfrac{1}{2} \int_{-\pi}^{0} x\,dx + \tfrac{1}{2} \int_{0}^{\pi} 2x\,dx$$

$$= \tfrac{1}{\pi} \left[\frac{x^2}{2} \right]_{-\pi}^{0} + \frac{2}{2\pi} \left[\frac{x^2}{2} \right]_{0}^{\pi}$$

$$= \tfrac{1}{\pi} \left[0 - \frac{\pi^2}{2} \right] + \frac{2}{\pi} \left[\frac{\pi^2}{2} - 0 \right]$$

$$a_0 = \frac{-\pi}{2} + \pi = \pi/2 \quad \text{—(1)}$$

$$a_x = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} (x \cos nx\, dx) + \frac{1}{\pi} \int_{0}^{\pi} (2x \cos nx)\, dx$$

$$= \frac{1}{\pi} \left[x \left(\frac{\sin nx}{n} \right) - \left(- \frac{\cos nx}{n^2} \right) \right]_{-\pi}^{0}$$

$$= + \tfrac{2}{\pi} \left[x \left(\frac{\sin nx}{n} \right) - \left(- \frac{\cos nx}{n^2} \right) \right]_{0}^{\pi}$$

$$an = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos nx}{n^2} + \frac{2}{\pi}\left(\frac{\cos^n x}{n^2} - \frac{\cos(0)}{n^2}\right]\right.$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

so

$$an = \begin{cases} \frac{-2}{\pi n^2} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases} \quad (2)$$

$$bx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ \sin nx\, dx = \frac{1}{\pi}\int_{-\pi}^{0} x \sin nx\, dx + \frac{2}{x}\int_{0}^{\pi} x \sin nx\, dx$$

$$= \frac{1}{\pi}\left\{x\left(\frac{-\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x - \frac{\cos n^2}{n} - \left(-\frac{\sin x}{n^2}\right)\right]_0^{\pi}$$

$$bn = \frac{1}{\pi}\left[-\frac{\pi \cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi \cos \pi}{n}\right. = \frac{-3\cos n x}{n} = \frac{3(-1)^{n+1}}{n}$$

The required forier series is

$$f(x) = \frac{a_0}{2} + \sum_{n+1}^{\infty} an \cos nx + bn \sin nx$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos(2x-1)x}{(2n-1)^2}$$

$$+ 3\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n}$$

Q3 Answer

Given

$$y'' - 4y' + 13y = 8\sin 3x$$

We have to find $y = y_c + y_P$

for y_c the characteristic (auxilary Eqn) is

$$m^2 - 4m + 13 = 0$$

$$= m = \frac{4 \pm \sqrt{16-52}}{2} \Rightarrow m = \frac{4+63}{2}$$

$\Rightarrow m = 2 + 3i$; $\alpha = 2$ & $\beta = 3$

So $y_c = e^{2x}\{C_1 \cos 3x + C_2 \sin 3x\}$

for y_p let

$$y_P = \text{Imag}\left(\frac{1}{m^2 - 4m + 13} \, 8e^{3ix}\right.$$

$$= 8\,\text{Imag} \frac{e^{3ix}}{(x)^2 - 4(3i) + 13}$$

$$= 8\,\text{Imag} \frac{e^{3ix}}{-9 - 12i + 13}$$

$$= 8\,\text{Imag} \frac{e^{3ix}}{4 - 12i}$$

$$yp = 2\ \text{Imag}\ \frac{e^{3ix}}{(1-3i)} \times \frac{(1+3i)}{1+3i}$$

$$yp = 2\ \text{Imag}\ \frac{(1+3i)\ (e^{3ix})}{(1)^2 - (3i)^2}$$

$$yp = 2\ \text{Imag}\ \frac{(1+3i)\ (e^{3ix})}{10}$$

$$yp = \frac{2}{10}\left(\text{Imag}\ (1+3i)\ (\cos x + i\ \sin 3x)\right)$$

$$yp = \frac{2}{10}\left(\sin 3x + 3\cos 3_2\right)$$

The gernel solution is

$$y = y_c + yp$$

$$y = c_1 e^{2x} \cos 3x + c_2 e^{2x} \sin 3x + \frac{2}{10}\left(\sin 3x + 3\cos 3x\right)$$

Now use the initial condition $y(0) = 1$

$$y(0) = c_1 e^{(0)} \cos(0) + c_2\ e^{(0)}\ \sin(0) + \frac{2}{10}\left(\sin(0) + 3\cos(0)\right)$$

$$c_1(1) + 0 + 0 + \frac{2}{10}\left(3(1)\right)$$

$$c_1 + \frac{6}{10} \Rightarrow c_1 = 1 - \frac{6}{10} = \frac{4}{10} = \frac{2}{5}$$

again use the another Initial condition

$$y'(0) = 2$$

So

$$y' = c_1\, 2e^{2x}\cos 3x + c_1 e^{2x}(-3\sin 3x) + c_2\, 2e^{24}\sin 3x + c_2 e^{24}(3\cos 3x) + 2/10\,(\cos 3x - 3\sin 3x$$

$$y'(0) = c_1 \underline{2} e^{(0)}\cos(0) + c_1 e^{(0)}(-3\sin(0)) + c_2\, 2e^{(0)}\sin(0) + c^2 e^{(0)}(3\cos(0) + 2/10\,(\cos(0) - 3(\sin(0)))$$

$$2 = 2c_1 + 0 + 0 + c_2\, 3(1) + 2/10\,(1 - 3(0)$$

$$2 = 2c_1 + 3c_2 + 2/10$$

$$\boxed{\text{use } c_1 = 2/5}$$

$$2 = 2(2/5) + 3c_2 + 2/10$$

$$1/3\,(2 - 4/5 - 2/10) = c_2$$

$$\Rightarrow c_2 = 1/3\left(\frac{2x - 8 - 2}{10}\right) = 1/3$$

So General solution is

$$y = 2/5\, e^{2x}\cos 3x + 1/3\, e^{2x}\sin 3x + 2/10\,[\sin 3x + 3\cos 3x].$$

b The required solution

Q4
Ans

$(D^2 - DD')z = \cos x \cos 2y$

auxilary equation is

$m^2 - m = 0 \implies m = 0 \;,\; m = 1$

Hence the complementry function is given by

$Zc = f_1(y) + f_2(y+x)$

for Particular INTEGRAL we have

$zP = \frac{1}{D^2 - DD'} \cos x \cos 2y$

$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} \left[\cos(x-2y) + \cos(x-2y)\right]$

$= \frac{1}{2}\left[\frac{1}{1+2}\cos(x+2y) + \frac{1}{-1-2}\cos(x-2y)\right]$

$= \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$

Hence the complete solution is given by

$$\boxed{z = f_1 y + f_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)}$$

Ans