

Q1:- Sol. Given that:

$$\begin{bmatrix} 1 & ID_2 & 3 & 0 & 5 \\ 0 & 1 & -ID_{last} & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID_2 \end{bmatrix}$$

$$\begin{matrix} ID_2 = 3, & ID_{last} = 5 \\ \begin{bmatrix} 1 & 3 & 3 & 0 & 5 \\ 0 & 1 & -5 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \end{matrix}$$

By using elementary row operations,

$$\underbrace{R_1 - 3R_2}_{\text{By using elementary row operations}} \begin{bmatrix} 1 & 3 & 3 & 0 & 5 & | & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -5 & 0 & 7 & | & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -6 & | & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & | & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{matrix} R_1 + 18R_3 \\ R_2 + 5R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 18 & 0 & -16 & | & 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & -5 & 0 & 7 & | & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -6 & | & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & | & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & +92 & | & 1 & -3 & -18 & 0 & 0 \\ 0 & 1 & 0 & 0 & -23 & | & 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 & -6 & | & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & | & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{pmatrix} 92 \\ -23 \\ -6 \\ 3 \end{pmatrix}$$

Q2:- (a) Sol:
$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & +2 & -5 & -1 \end{bmatrix}$$

using elementary row operations,

$$R_3 - 2R_2 \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & +3 & -5 \end{bmatrix}$$

Similarly for

$$R_3 + 2R_2 \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & +3 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

So $R_3 - 2R_2$ is an inverse row operation of $R_3 + 2R_2$ -

(b) Sol: (a).
$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

Yes, it is an echelon form of matrix because each pivot value below and above zero entry exists -

(b).
$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Sol: No, it is not an echelon form of matrix because in 3rd row there is not pivot value - exist -

$$(c) \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

No, it's not a reduced echelon matrix because each pivot value must be equal to 1.

$$(d) \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

No, because in 2nd row there is no pivot value exists.

Q3 (a) Sol: Row echelon form is achieved when one of the vectors is perfectly aligned with a declared axis. Reduced row echelon form is achieved when all the vectors are perfectly aligned with a declared axis.

$$(b) \text{ Sol: } \left[\begin{array}{ccc|ccc} 1 & 1 & 8 & 1 & 0 & 0 \\ 2 & 8 & -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 1 & -4 & 1 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 + R_1 \\ R_4 - R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 8 & 1 & 0 & 0 \\ 0 & 6 & -17 & -2 & 1 & 0 \\ 0 & 1 & 8 & 1 & 0 & 1 \\ 0 & -5 & -7 & 1 & 0 & 0 \end{array} \right]$$

$$\begin{array}{l} R_1 - R_3 \\ -5R_4 + R_3 \\ 6R_3 - R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 6 & -17 & -2 & 1 & 0 \\ 0 & 1 & 8 & 1 & 0 & 1 \\ 0 & 0 & 43 & -4 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_4 \\ 43 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 6 & -17 & -2 & 1 & 0 \\ 0 & 0 & 65 & 8 & -1 & 6 \\ 0 & 0 & 43 & -4 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & 0 \\ 0 & 6 & -12 & -2 & 1 \\ 0 & 0 & 65 & 8 & -1 \\ 0 & 0 & 1 & -4 & 0 \end{array} \right] \begin{array}{l} \\ \\ \\ \frac{1}{43} \end{array}$$

calculate from

~~IP₁~~ $IP_2 = 0$

$IP_3 = 8$

$IP - \text{First-Last} = \frac{1}{43} \quad \underline{\text{Ans}}$