M☐T☐W☐T☐F☐S

H/W☐-C/W☐ Dated:........./........./20.......

Name: M Kamran

IDNO : 7888

Semester : 6^{th}

Question: 1

Define Drag with its components, write down the equation for friction drag coefficient both in laminar & turbulent boundary layer.

Ans:

Force on immense Body:

A body in whole immense in a homogenous fluid may be subject to two kind of forces arising from relative motions b/w body and fluid. These forces are termed as drag & lift depending whether force is parallel or at right angle in motion.

Drag forces on submerged body can have two components.

Pressure Drag Force:

It is equal to the integration of component in the direction of motion of all pressure forces enteveted on the surface body.

$F_p = C_p \rho \frac{v^2}{2} A$ C_p: depend on shap

Friction Drag: It is equal to integration of components shear stress along the surface of the body in duction of motion.

$F_f = C_f \rho \frac{v^2}{2} BL$ C_f: depend on viscosity

Friction Drag of Boundary layer:

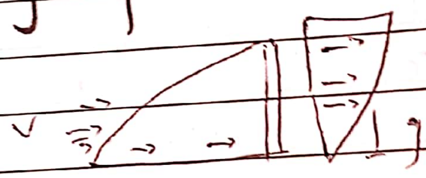
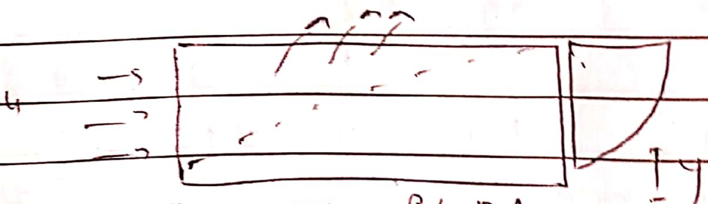


Figure show growth of the boundary layer along one side of smooth plate is steady from is compressible fluid consider in control volume.



$F_f = \int \tau_0 B dx$

where δ is distance from boundary layer to plate.

Now $= F_x =$ drag $=$ rate of momentum in $x =$ direction leaving through BC

$\Rightarrow$ rate of momentum in x- direction leaving through AB - moment in x direction leaving along OA

Impulse Moment Principle.

$$\Sigma F = \frac{d(mv)}{dt} = \frac{\rho \times vd}{dt} \times v = \rho Q V$$

$$\Sigma F_x = \rho \, Q_2 V_2 - Q_1 V_1$$

$DA \rightarrow \rho v \, (V \times B \times \delta)$

$CB \rightarrow \rho B \int^{B} u^2 dy$

$AB \rightarrow \rho \, (V \times B \times \delta - B \int^{B} u \, dy) v$

Putting value

$$F_x = \rho B \int^{\delta} u \, (U - u) \, dy$$

Solving this equation

$F_x = \rho B V^2 \delta \alpha$ where α the function of boundary layer velocity condition distribution

Now to find local wall shear stress

$$\tau = \frac{F_x}{Area} = \tau_0 = \frac{dF_x}{B\,dx}$$

$$F_x = \rho B V^2 \delta\,\alpha$$

$$F_x = \rho v^2 \alpha \frac{d\delta}{dx}$$

$$\tau_0 = \rho V^2 \alpha \frac{d\delta}{dx}$$

Laminar Boundary layer

In case of laminar

$$\tau_0 = \mu \left(\frac{dx}{dy}\right)_{y=0} \quad \frac{\mu}{\delta}\left(\frac{du}{dx}\right) = \frac{\mu v}{\delta}\left[\frac{dF(h)}{d\eta}\right]$$

By solving

$$\tau_0 = \frac{\mu u B}{\delta} \quad \text{--(1)}$$

Equation $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

$$\delta\, d\delta = \frac{\mu B}{\rho u \alpha}\, dx$$

Solving eq

$$\frac{\delta^2}{2} = \frac{\mu B x}{\rho u \alpha} + C$$

At $x=0$, $\delta=0$, $c=0$

$$\delta = \sqrt{\frac{2\mu B}{\rho u \alpha}} = \sqrt{\frac{2\beta}{\alpha}} \cdot \frac{x}{\sqrt{R_x}} \qquad R_x = \frac{x u \rho}{\mu}$$

Experimentally

$\beta = 1.63$, $\alpha = 0.135$

$$\frac{\delta}{x} = \sqrt{\frac{2\times 1.63}{0.135}} \times \frac{1}{\sqrt{R_x}} = \frac{4.91}{\sqrt{R_x}}$$

where R_x may be called the load reynolds number. It should be noted that R_x increase linearly in down stream direction.

Now

$$F_f = B\int_0^L \tau_0 \, dx$$

$$\tau_0 = 0.332 \frac{\mu v}{x} \sqrt{R_x}$$

$$R_x = \frac{x v \rho}{\mu}$$

Thus

$$F_f = 0.664 B \sqrt{\rho \mu L U^3}$$

$$F_f = C_F \frac{\rho v^2}{2} BL$$

$$C_F = 1.328\sqrt{\frac{\mu}{\rho L V}} = \frac{1.328}{\sqrt{R}}$$

where R is based on characteristics left lenght of whole plate. The laminar boundary layer will remain lemiar if Rn in of about 500,000

Part, B.

Derive equation for critical depth, critical velocity of rectangular section of a channel?

Ans:

Specific Energy: defined as the energy head reffered to channel bed to datum.

$$E = y + \frac{v^2}{2g}$$

If channel is uniform depth and releatively wide, flow near center of

Channel will be unaffected by side boundaries

Flow q per unit width

$$q = Q/b$$

Now average velocity

$$V = \frac{Q}{A} = \frac{q\cancel{b}}{\cancel{b}y} = q/y$$

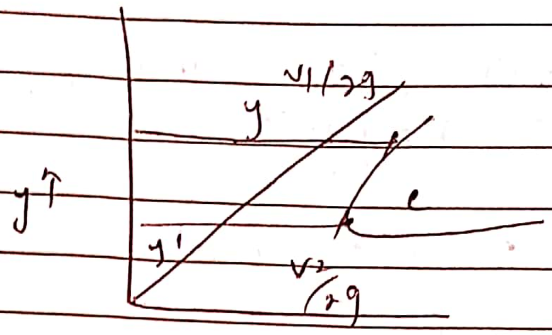
Thus $E = y + \frac{1}{g}\left(\frac{q^2}{y^2}\right)$

Consider, how E will vary with y if q remains constant

$$(E-y) = \frac{q^2}{2g(y^2)}$$

$$(E-y)y^2 = \frac{q^2}{2g} = \text{constant}$$

Plot of E vs y is parabala



q = constant

This specific energy diagram

From particular q, there will be two kind of possible values of y for given E.
The equation is cubic with three roots with third root being negative giving no value.

Thus two alternate depth represent two totally different flow regimes. slow and deep portion and fast & shallow or lower portion.
Point divide b/w two regimes of flow.

The relation of critical depth can found as

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

minimum spec energy $\frac{dE}{dy} = 0$

$$\frac{dE}{dy} = 1 + \frac{\cancel{2}}{\cancel{2}g}\left(\frac{q^2}{y^2}\right)$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} \Rightarrow 0$$

$$1 = \frac{q^2}{gy^3} \Rightarrow q^2 = gy^3 \text{ or } \frac{q^2}{g} = y_{cr}^3$$

$$q^2 = gy^3 \Rightarrow y_{cr}\left(\frac{q^2}{g}\right)^{1/3}$$

$$q = Vy \;:-\; V_c^2 = gy_3$$

$$\boxed{y_c = \frac{V_c^2}{g}}$$

$$E_{min} = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2}$$

$$= \frac{3}{2} y_c$$

Question: 2

Find the depth of rectangular depth if water flows at the rate of 3.5 m^3/s with bed slope of 0.0008 and n = 0.0219 width of bed in your student ID number.
Also find critical depth, critical velocity.

Is flow subcritical or super critical?

Given data

$Q = 3.5\ m^3/s$
$S_o = 0.0008$
$n = 0.219$
$b = 7888\ mm$

Required.

$y = ?$

$y_{cr} = ?$

$V_{cr} = ?$

$$Q = \frac{1}{n} \cdot A \; R_h^{2/3} \cdot S_o^{1/2}$$

$$A = y \times b$$

$$= y \times 7.888$$

$$R_h = \frac{A}{P}$$

$$= \frac{7888}{2y + 15.694}$$

Putting values

$$3.5 = \frac{1}{0.0219} \times 7888 \times \left(\frac{7888}{2y + 15.764}\right)(0.0008)^{1/2}$$

$$y = 0.7222 \, m$$

$$y = 722.8 \, mm$$

Now

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$q = \frac{Q}{b} = \frac{3.5}{7888} = 0.446 \, m^2/s$$

$$y_{cr} = 0.2726 \, m$$

$$y_{cr} = 272.6 \, mm$$

$V_{cr} = ?$

$$V_{cr} = \sqrt{g y_{cr}}$$

$$V_{cr} = \sqrt{9.81 \times 0.272}$$

$$V_{cr} = 1.63\ m/s$$

Since

$$= y > y_{cr}$$

Flow is subcritical

No 3:

Find friction drag on one side of a smoth plate with 200mm wide and 800mm legth placed specific gravity of 0.89. the undisturbed velocisty is 5m/s and K.t velocity is $0.93 \times 10^{-4}\ m^2/s$?

Given Data.

wide = 200m

L = 800 mm

Specific gravity = 0.89

$U = 5$

$V = 0.93 \times 10^{-4}\ m^2/s$

Solution

$$R = \frac{LU}{V}$$

$V = 0.93 \times 10^{-4}\ m^2/s$

$L = 0.80$

$U = C$

Put values

$$\frac{0.80 \times 5}{0.93 \times 10^{-4}}$$

$= 43010 < 500,000$

Thus

$$C_f = \frac{1.328}{\sqrt{R}}$$

$$\Rightarrow \frac{1.328}{\sqrt{43010}} = 0.0064$$

$$F_f = C_f\, \rho \frac{V^2}{2} \times BL$$

$$F_f = 0.006 \times 0.89 \times 1000 \times \frac{(5)^2}{2} \times 0.20 \times 0.80$$

$\Rightarrow$ 530.4

To find thickness of Boundary layer.

$$\frac{\delta}{x} = \frac{4.91}{\sqrt{R_x}} \quad \text{at } x = L$$

$$\delta = \frac{4.91}{\sqrt{43010}} \times 80\,cm$$

$$\delta = 1.89\,cm$$

$$F_f = 0.664 \times 8 \sqrt{\delta\, v \cdot L V^3}$$

$$0.664 \times 0.20 \sqrt{0.89 \times 1000 \times 1000 \times 0.39 \times 0.934 \times 10 \times 0.8 \times (5)^3}$$

$$\boxed{F_f = 11.39N}$$ Answer