


Cours: Linear Algebra

Module: Summer Semester

Instructor: Sir. Himayat Ullah

Name: Farhan Ali

ID: 14873

Signature: 

Page = 1

Q3:-

Find A^{-1} where $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$

Solution:-

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix}$$

$$= 3 \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} - 2 \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1 \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3(-4-6) + 2(-15-2) + 1(0-6)$$

$$|A| = -94$$

Now

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} = -18 \quad A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$\text{Prise} = 2$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = -2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 0 & 2 \end{vmatrix} = -10, \quad A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 0 \end{vmatrix} = 28$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

Page = 3

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-94} \begin{pmatrix} 18 & 6 & 10 \\ -17 & 10 & 1 \\ 6 & 2 & -28 \end{pmatrix}$$

Ans.

Q1

part (a) :-

$$\text{let } A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$$

Identify the (3,2) entry of AB.

Sol:-

Identify (3,2) = ?

Row₃(A) · Column₂(B)

$$\Rightarrow [0 \ 1 \ -2] \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$

$$= (0)(4) + (1)(-1) + (-2)(2)$$

$$= 0 + (-1) + (-4)$$

$$= -1 - 4$$

$$= \boxed{-5} \text{ Ans}$$

Q1

Part (B)

Find the quadratic polynomial that interpolates the points

$$(1, 3), (2, 4), (3, 7)$$

Solⁿ

$$\text{As } a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$

$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$

$$a_2 x_3^2 + a_1 x_3 + a_0 = y_3$$

$$\text{Now } (x_1, y_1) = (1, 3), (x_2, y_2) = (2, 4)$$

$$(x_3, y_3) = (3, 7) \quad \text{put in eqn}$$

$$a_2 + a_1 + a_0 = 3$$

$$4a_2 + 2a_1 + a_0 = 4$$

$$9a_2 + 3a_1 + a_0 = 7$$

$$\text{As } \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 7 \end{array} \right]$$

$$\xrightarrow{R} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -20 \end{array} \right] \begin{array}{l} R_2 - 4R_1 \\ R_3 - 9R_1 \end{array}$$

$$\xrightarrow{R} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 4 \end{array} \right] R_3 - 3R_2$$

$$\text{So } a_2 + a_1 + a_0 = 3 \quad \text{--- (1)}$$

$$-2a_1 - 3a_0 = -8 \quad \text{--- (2)}$$

$$a_0 = 4 \quad \text{--- put in (2)}$$

$$-2a_1 - 12 = -8$$

$$\Rightarrow a_1 = \frac{4}{-2} = -2 \quad \text{put in (1)}$$

$$a_2 - 2 + 4 = 3 \Rightarrow a_2 = 1$$

Q 28-

if A and B are $n \times n$ matrices where $|A| = 2$ and $|B| = -3$ Calculate $|A^{-1} B^T|$

Solution

$$|A^{-1} B^T|$$

$$= |A^{-1}| |B^T|$$

$$= \frac{1}{|A|} |B|$$

$$|B^T| = |B|$$

$$\text{So } |A^{-1} B^T| = \frac{1}{|A|} |B|$$

$$= \frac{1}{2} \cdot 3$$

$$= \frac{3}{2}$$

Ans.

Q 2

Part (B) :-

$$x + y + 2z = 1$$

$$x - 2y + z = -5$$

$$3x + y + z = 3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & -2 & -5 & 0 \end{array} \right] \begin{array}{l} R_2 - R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 7 \\ 0 & 1 & 7/3 & 2 \\ 0 & -2 & -5 & 0 \end{array} \right] \begin{array}{l} R_2 \times 3 \\ R_3 + 2R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 7/3 & 2 \\ 0 & -2 & -13/2 & 4 \end{array} \right] R_3 + 2R_2$$

Page = 9

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & \frac{1}{3} & 2 \\ 0 & 0 & 1 & -\frac{8}{13} \end{array} \right] R_3 \times \frac{2}{-13}$$

$$x + y + 2z = 1 \quad \text{--- (i)}$$

$$y + \frac{1}{3}z = 2 \quad \text{--- (ii)}$$

$$z = -\frac{8}{13} \quad \text{--- (iii)}$$

Now put eq (iii) in (i)

$$y + \frac{2}{3} \times \frac{-8}{13} = 2$$

$$y - \frac{8}{39} = 2$$

$$y = 2 + \frac{8}{39}$$

$$y = \frac{78+8}{39} = \frac{86}{39}$$

Now put values of y in (i)

Page = 10

$$n + \frac{86}{39} + 2\left(\frac{-8}{13}\right) = 1$$

$$n + \frac{86}{39} - \frac{16}{13} = 1$$

$$n + \frac{38}{39} = 1$$

$$n = 1 - \frac{38}{39}$$

$$n = \frac{1}{39}$$