NAME: MAHAM KHALID

I-D: 15016

DEPARTMENT: MICROBIOLOGY

PAPER: MATHEMATICS

FINAL - SUMMER EXAM

QUESTION: 1

a) Solve the system with two variables by Cramer's rule;

(i) $X - 2y = 1$

$3X + y = 10$

Solution:-

In terms of matrices we can write the above system as;

$$\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \end{bmatrix} \Rightarrow AX = B.$$

where $A = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$.

$|A| = [X] - 3(-2) = 1 + 6 = 7 \neq 0$

Replacing the co-efficients of x in A (that is 1,3) by 1,10 of B to find A.

$A_2 = \begin{bmatrix} 1 & -2 \\ 10 & 1 \end{bmatrix}$ giving $|A| = |x| - 10(-2) = 1 + 20 = 21$

$$\frac{|A_2|}{A} = \frac{21}{7} = 3$$

For A_y = replace the coefficients of y in A (that is -2,1) by 1,10 of B.

Then, $A_y = \begin{bmatrix} 1 & 1 \\ 3 & 10 \end{bmatrix}$ giving $|A_y| = |x\ 10 - 3(1) =$

$10 - 3 = 7$

$$\therefore y = \frac{|A|}{A} = \frac{7}{7} = 1$$

Hence the following solution set = (3, 1)

Answer (3, 1)

Q.16) Solve the system with two variables by inversion method.

$x - 3y = 0$

$2x + y = 7$

Solution:-

$x - 3y = 0$

$2x + y = 7$

$$\begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \end{bmatrix}$$

$$A \qquad X \qquad B$$

By Inversion Method,

$$X = A^{-1}B \quad \text{—} (1)$$

$$A = \begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}$$

$$|A| = (1 \times 1) - (-3)(2) = 1 + 6 = 7$$

$$\text{adj } A = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \text{adj } A / |A| = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix} \quad \text{—} (2)$$

Putting 2 in 1

$$X = \frac{1}{7} \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 7 \end{bmatrix}$$

$$X = \frac{1}{7} \begin{bmatrix} 1 \times 0 + 3 \times 7 \\ -2 \times 0 + 1 \times 7 \end{bmatrix}$$

$$X = \frac{1}{7} \begin{bmatrix} 0 + 21 \\ 0 \; 7 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 21 \\ 7 \end{bmatrix}$$

$$X = \begin{bmatrix} 21/7 \\ 7/7 \end{bmatrix} \Rightarrow \boxed{\begin{matrix} x = 3 \\ y = 1 \end{matrix} \leftarrow \text{Ans}}$$

Q. 2: Solve the following Quadratic Equation by using Factorization Method.

a) $4y^2 + 15y + 6 = 4Y$

Solution:-

$$4y^2 + 15y + 6 = 4y$$

$4y^2 + 15y - 4y + 6 = 0$

$4y^2 + 11y + 6 = 0$

$4y^2 + 8y + 3y + 6 = 0$

$4y(y+2) + 3(y+2) = 0$

$(4y+3)(y+2) = 0$

$4y + 3 = 0$, $y + 2 = 0$

$4y = -3$

$\boxed{y = -3/4}$ Ans

$\boxed{y = -2}$ Ans

b) $x^2 + 15x = -50$

Solution:-

$x^2 + 15x = -50$

$x^2 + 15x + 50 = 0$

$x^2 + 10x + 5x + 50 = 0$

$x(x+10) + 5(x+10) = 0$

$(x+5)(x+10) = 0$

$x + 5 = 0$; $x + 10 = 0$

$\boxed{x = -5}$; $\boxed{x = -10}$

c) $y^2 = 6y + 27$

Solution :-

$\Rightarrow y^2 = 6y + 27$

$\Rightarrow y^2 - 6y - 27 = 0$

$\Rightarrow y^2 - 9y + 3y - 27 = 0$

$\Rightarrow y(y-9) + 3(y-9) = 0$

$\Rightarrow y + 3 = 0$; $y - 9 = 0$

$\Rightarrow \boxed{y = -3}$; $\boxed{y = 9}$ Ans

(Q.3) Solve the following Quadratic Equations by Using Factorization Method.

a) The sum of two numbers is 27 & their Product is 50. Find the numbers.

Solution:-

Let the numbers be x & y

$$x + y = 27 \quad \text{—} ①$$
$$xy = 50 \quad \text{—} ②$$

From equation ①

$$x + y = 27$$
$$x = 27 - y \quad \text{—} ③$$

Putting ③ in ②.

$$(27-y)(y) = 50$$
$$27y - y^2 = 50$$
$$y^2 - 27y + 50 = 0$$
$$y^2 - 25y - 2y + 50 = 0$$
$$y(y-25) - 2(y-25) = 0$$
$$(y-2)(y-25) = 0$$
$$y - 2 = 0 \; ; \; y - 25 = 0$$
$$y = 2 \; ; \; y = 25$$

Putting $y = 2$ in ③

$$x = 27 - 2$$
$$x = 25$$

Putting $y = 25$ in ③

$$x = 27 - 25$$
$$x = 2$$

The numbers are 25 & 2

Q35) The three sides of a right angled triangle are x, $x+1$ & 5. Find x & the area, if the longest side is 5.

Solution:-

let, hyp = 5
Base = x
Perp = $x+1$

So, Pythagoraan theorum

$(hyp)^2 = (Base)^2 + (Perp)^2$

$(5)^2 = x^2 + (x+1)^2$

$25 = x^2 + x^2 + 2x + 1$

$25 = 2x^2 + 2x + 1$

$2x^2 + 2x + 1 - 25 = 0$

$2x^2 + 2x - 24 = 0$

$x^2 + x - 12 = 0$

$x^2 + 4x - 3x - 12 = 0$

$x(x+4) - 3(x+4) = 0$

$(x-3)(x+4) = 0$

$x - 3 = 0$; $x + 4 = 0$

$x = 3$; $x = -4$

Taking positive value of 3

So, base = $x = 3$

Perp = $x+1 = 3+4 = 4$

Now, Area = $1/2$ (base) (perp).

$= 1/2\,(3)\,(4)$

Area = 6 Ans.