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Mid Term Assignment

Multivariate Calculus

Q1:- Let $P(1,0,-3)$, $Q(0,-2,-4)$ and $R(4,1,6)$ be points.
(a) Find the equation of the plane through the points P, Q and R.
(b) Find the area of the triangle with vertices P, Q and R. Solution.

Answer:-

Part (a)

Solution:

The vector $\overrightarrow{PQ} \times \overrightarrow{PR} = \cancel{\langle -1, 0, 3 \rangle \times (}$

$$\overrightarrow{PQ} = \langle (0-1), (-2-0), (-4+3) \rangle$$
$$= \langle -1, -2, -1 \rangle$$
$$\overrightarrow{PR} = \langle (4-1), (1-0), (6+3) \rangle$$
$$= \langle 3, 1, 9 \rangle$$

$$\overrightarrow{PQ} \times PR = \langle -1, -2, -1 \rangle \times \langle 3, 1, 9 \rangle$$

$$= \begin{vmatrix} i & j & k \\ -1 & -2 & -1 \\ 3 & 1 & 9 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & -1 \\ 1 & 9 \end{vmatrix} \bar{i} - \begin{vmatrix} -1 & -1 \\ 3 & 9 \end{vmatrix} \bar{j} + \begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix} \bar{k}$$

$$= (-18+1) - (-9+3) + (-1+6)$$

$= \langle -17, 6, 5 \rangle$ is the normal vector of this plane, so equation of the plane

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is $-17(x-1)+6(y-0)+5(z+3)=0$

which simplifies to $17x-6y-5z=32$

Part (b)

Area $= \frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right| = \frac{1}{2}\left|\langle -1,-2,-1\rangle \times \langle 3,1,9\rangle\right|$

$= \frac{\sqrt{350}}{2}$

Answer:

Q2:- Let $f(x,y) = (x-y)^3 + 2xy + x^2 - y$.

Find the linear approximation $L(x,y)$ near the point $(1,2)$

Answers:-

Solution:-

$f_x = 3x^2 - 6xy + 3y^2 + 2y + 2x$ and

$f_y = -3x^2 + 6xy - 3y^2 + 2x - 1$, So

$f_x(1,2) = 9$ and $f_y(1,2) = -2$.

Then the linear approximation of f at $(1,2)$ is given by

$L(x,y) = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2) = 2 + 9(x-1) + (-2)(y-2)$.

Answers:

Q3:- Find the distance between the parallel planes $x+2y-z=-1$ and $3x+6y-3z=3$.

Answer:-

Solutions:

First we prove that the planes are parallel to eachother.

So: $x+2y-z=-1$
$3x+6y-3z=3$

$\langle 1, 2, -1\rangle$
$\langle 3, 6, -3\rangle$

$$\frac{1}{3} = \frac{\cancel{2}}{3\cancel{6}} = \frac{+1}{+3} = \frac{1}{3} = \frac{1}{3} = \frac{1}{3}$$

So prove that the two planes are parallel.

formula: $D = \dfrac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$.

Use a point from the second plane (for example) $(1,0,0)$) as (x_0, y_0, z_0) and the coefficents from the first plane $a=1$, $b=2$, $c=-1$, and $d=1$. We Compute $D = \dfrac{|1\cdot1+2\cdot0+(-1)\cdot0+1|}{\sqrt{1^2+2^2+(-1)^2}} = \dfrac{2}{\sqrt{6}}$

Ans...

Qu:- Find the following limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy + y^2}{x^2 + y^2}$$

Answer:

Solution:-

First, we will use the path $y = x$. Along this path we have,

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy + y^2}{x^2 + y^2} = \lim_{(x,x)\to(0,0)} \frac{x^2 - x^2 + x^2}{x^2 + x^2}$$

$$= \lim_{(x,x)\to(0,0)} \frac{x^2}{2x^2} = 1/2$$

Now, let's try the path $y = 0$.

Along this path the limit becomes,

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy + y^2}{x^2 + y^2} = \lim_{(x,0)\to(0,0)} \frac{x^2}{x^2} = 1$$

We have two paths that give different values for the given limit and so the limit doesn't exist.

Q5:- Find the equation of the tangent plane to the surface $z = 4x^3y^2 + 2y$ at point $(1, -2, 12)$.

Answer:-

Solution:-

Since $f(x, y) = 4x^3y^2 + 2y$, we have $f_x(x, y) = 12x^2y^2$ and $f_y(x, y) = 8x^3y + 2$.

Now plug in $x = 1$ and $y = -2$, we obtain ~~$f(x, y) = 12$~~ $f_x(1, -2) = 48$ $f_y(1, -2) = -14$.

Thus, the tangent plane has normal vector $n = \langle 48, -14, -1 \rangle$ at $(1, 2, 12)$ and the equation of the tangent plane is given by $48(x-1) - 14(y-(-2)) - (z-12) = 0$
Simplifying, we obtain $48x - 14y - z = \boxed{64}$

Ans

Q6:- Find all the second order partial derivatives for $f(x,y) = \sin(2x) - x^2e^{3y} + y^2$.

Answer:-

Solution:-

$f_x = 2\cos(2x) - 2xe^{3y}$, so f_{xx}

$= -4\sin(2x) - 2e^{3y}$ and $f_{xy} = -6xe^{3y}$. f_y

$= -3x^2e^{3y} + 2y$, so $f_{yy} = -9x^2e^{3y} + 2$, and
$f_{yx} = \boxed{-6xe^{3y}}$. thus $f_{xy} = f_{yx}$.

Ans:-