

Solution

The moment of Section A is

$$M = 11000 \times 15 = 165,000$$

and the torque on the shaft is

$$T = 11000 \times 0.15 = 1650$$

The normal stress due to M at A is

$$\sigma = \frac{64md}{\pi d^4} = \frac{32M}{\pi d^3}$$

Maximum Shear Stress due to T at A is

$$\tau = \frac{32Td}{\pi d^4} = \frac{16T}{\pi d^3}$$

The Shear Stress due to shear force is zero at A

$$\sigma_{ns} = \frac{1}{2} \sigma \pm \frac{1}{2} (\sigma^2 + 4\tau^2)^{1/2}$$

Maximum Shear Stress theory

$$\tau_{max} = \frac{1}{2} (\sigma_1 - \sigma_2)$$

$$\tau_{max} = \frac{1}{2} \frac{32}{\pi d^3} (M^2 + T^2)^{1/2}$$

$$= \frac{16}{\pi d^3} (M^2 + T^2)^{1/2}$$

$$= \frac{16}{\pi d^3} (165000^2 + 1650^2)^{1/2}$$

$$= \frac{2640131.99}{\pi d^3} = \frac{840806.36}{d^3}$$

$$N_{\text{time}} = \frac{840806.36}{d^3} \text{ Pa}$$

This should not exceed the maximum ~~the~~ Shear Stress value at yielding in uniaxial tension test
This

$$\frac{1}{d^3} 840806.36 = \frac{\sigma_y}{2}$$

$$\frac{1}{d^3} 840806.36 = \frac{207 \times 10^{-6}}{2} = 103.5 \times 10^{-6}$$

$$d^3 = \frac{840806.36}{103.5} \times 10^{-6} = 8123.73 \times 10^{-6}$$

$$d = 20.10 \times 10^{-6}$$

$$d = 20.1 \text{ cm}$$

ii) Octahedral Shear Stress theory

$$T = \frac{1}{3} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

with $\sigma_2 = 0$

$$T = \frac{1}{3} [2\sigma_1^2 + 2\sigma_3^2 - 2\sigma_1\sigma_3]^{1/2}$$

$$T = \frac{\sqrt{2}}{3} (\sigma^2 + 3\tau^2)^{1/2}$$

$$T = \frac{\sqrt{2}}{3 \pi d^3} (4M^2 + 3T^2)^{1/2}$$

$$T = \frac{\sqrt{2}}{3 \pi d^3} [4(165000)^2 + 3(1650)^2]^{1/2}$$

$$T = \frac{\sqrt{2}}{3 \pi d^3} [330012.37]$$

$$T = \frac{\sqrt{2}}{3 \pi} \sigma_y$$

Equating this Octahedral Shear Stress at yielding of an uniaxial tension bar & using factor = 15

$$\frac{\sqrt{15}}{3} = \sigma_y$$

$$\Rightarrow 15 \times 330012.37 = \pi d^3 \sigma_y$$

$$d^3 = 4950.13 \times 10^{-6}$$

$$d = 1704.27 \times 10^{-6}$$

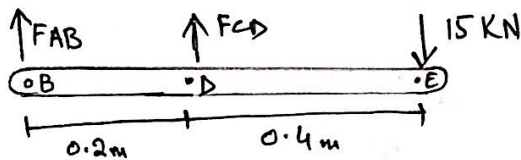
$$d = 1704.27 \text{ } \mu\text{m}$$

Q No 2

Solution

- Apply a free body analysis to the bar BDE to find the forces exerted by link AB & DC
- Evaluate the deformation of links AB & DC or the displacements of B and D
- Work out the geometry to find the deflection at E, given the deflection at B and D

Free body: Bar BDE



$$\sum M_B = 0$$

$$0 = -(15 \times 0.6 \text{ m}) + F_{CD} \times 0.2 \text{ m}$$

$$F_{CD} = 45 \text{ kN (tension)}$$

$$\sum M_D = 0$$

$$0 = -(15 \times 0.4) - F_{AB} \times 0.2 \text{ m}$$

$$F_{AB} = -30 \text{ kN (compression)}$$

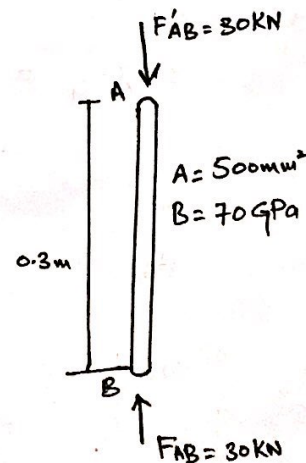
Displacement of B

$$\delta_B = \frac{PL}{AE}$$

$$\delta_B = \frac{(-30 \times 10^3 \text{ N})(0.3 \text{ m})}{(500 \times 10^{-6} \text{ m}^2)(70 \times 10^9 \text{ Pa})}$$

$$\delta_B = -2.57 \times 10^{-4}$$

$$\delta_B = 0.25 \uparrow$$



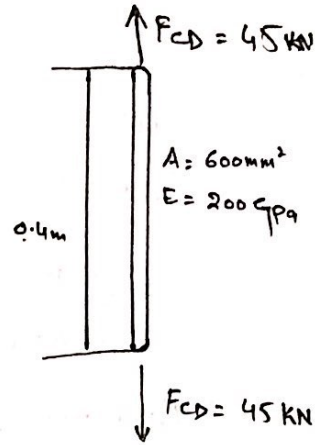
Displacement of D

$$\delta_D = \frac{PL}{AE}$$

$$\delta_D = \frac{(45 \times 10^3 \text{ N})(0.4 \text{ m})}{(600 \times 10^{-6} \text{ m}^2)(200 \times 10^9 \text{ Pa})}$$

$$\delta_D = 1.5 \times 10^{-4}$$

$$\delta_D = 0.15 \text{ mm} \downarrow$$



Displacement of D

$$\frac{BB'}{DD'} = \frac{BH}{HD}$$

$$\frac{0.25}{0.15} = \frac{(200 \text{ mm}) - x}{x}$$

$$0.25x = (200 - x)0.15$$

$$0.25x = 30 - 0.15x$$

$$0.25 + 0.15x = 30$$

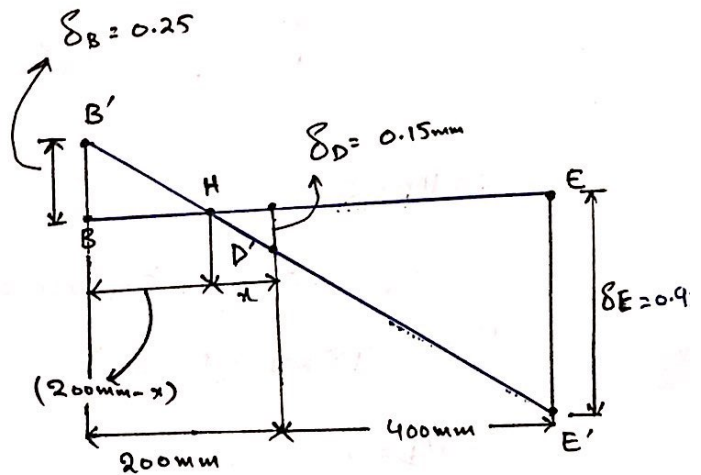
$$0.4x = 30$$

$$x = 30 / 0.4 = \underline{\underline{75}}$$

$$\frac{EE'}{DD'} = \frac{HE}{HD}$$

$$\frac{\delta_E}{0.15} = \frac{400 + 75}{75}$$

$$\delta_E = 0.95$$



Q No 3

Given Data :

$$G = 15 \times 10^6$$

Allowable Shearing Stress = 10 ksi

$$x = 15 + 10$$

$$x = 25$$

Solution:

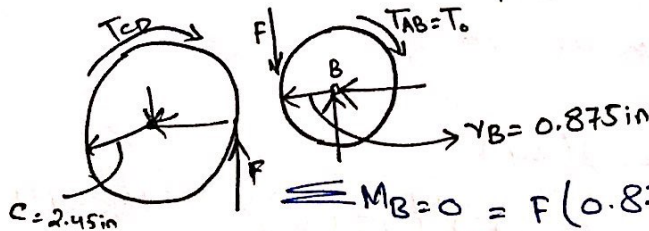
1* Apply a static equilibrium analysis on the two shafts to find a relationship b/w T_{CD} & T_0

2* Apply a kinematic analysis to relate the angular rotations of the gears

3* Find the maximum allowable torque on each shaft - choose the smallest

4* Find the corresponding angle of twist for each shaft & the net angular rotation of end A

1* Apply a static Equilibrium analysis on the two shafts to find a relationship b/w T_{CD} & T_0



$$\sum M_B = 0 = F(0.875 \text{ in}) - T_0$$

$$\sum M_C = 0 = F(2.45 \text{ in}) - T_{CD}$$

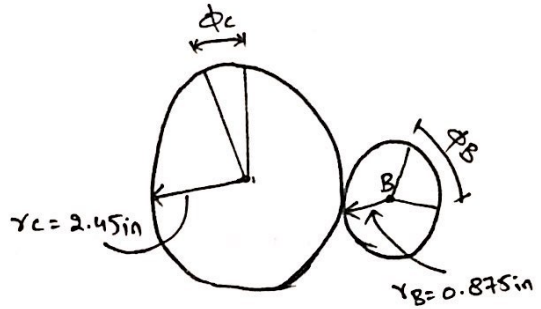
$$T_{CD} = 2.8 T_0$$

2* Apply a kinematic analysis to relate the angular rotations of the gears

$$r_B \phi_B = r_C \phi_C$$

$$\phi_B = \frac{r_C}{r_B} \phi_C = \frac{2.45 \text{ in}}{0.875 \text{ in}} \phi_C$$

$$\phi_B = 2.8 \phi_C$$



3* Find the T_0 for the max allowable torque on each shaft - Choose the smallest

$$\tau_{max} = \frac{T_{AB} L}{J_{AB}}$$

~~10000~~

$$10000 \text{ psi} = \frac{T_0 (0.375 \text{ in})}{\frac{\pi}{2} (0.375 \text{ in})^4}$$

$$\underline{T_0 = 827.9}$$

$$\tau_{max} = \frac{T_{CD} L}{J_{CD}} \quad 10000 \text{ psi} = \frac{2.8 T_0 (0.5 \text{ in})}{\frac{\pi}{2} (0.5 \text{ in})^4}$$

$$\frac{10000 \times 0.0981}{2.8 \times 0.5} \Rightarrow T_0 = \frac{981.25}{1.4} = 700 \text{ lb.in}$$

4* Find the corresponding angle of twist for each shaft & the net angular rotation of end A

$$\phi_{A/B} = \frac{T_{AB} L}{J_{AB} G} = \frac{(700)(24)}{\frac{\pi}{2} (0.375)^4 (25 \times 10^6)} = \frac{16800}{1.5710 \times 0.0197 \times 25 \times 10^6}$$

$$\phi_{A/B} = \frac{16800}{773255} = 0.0217 \text{ radians}$$

$$\phi_{A/B} = 1.24^\circ$$

$$\phi_{C/D} = \frac{T_{CDL}}{J_{CD^G}} = \frac{2.8 \times 700 \times 24}{\frac{\pi}{2} (0.5 \text{ in})^4 (25 \times 10^6)}$$

$$= \frac{47040}{1.57 \times 0.0625 \times 25 \times 10^6} = 0.019 \text{ rad} = 1.08^\circ$$

$$\phi_B = 2.8 \phi_C = 2.8 \times 1.08 = 3.024$$

$$\phi_A = \phi_B + \phi_{A/B} = 3.024 + 1.24^\circ = 4.264$$



Q No 4: Determine the location for the Shear Centre of the channel section

~~Solution~~ Given Data:

$$b = 4 \text{ inches}$$

$$h = 6 \text{ in}$$

$$\text{and } t = 0.15 \text{ in}$$

Req: Determine the Shear stress distribution for $V = 15 + 3 \text{ kips}$
 $V = 18$

Solution

$$e = \frac{FH}{I}$$

* Where

$$F = \int_0^b q ds = \int_0^b \frac{VA}{I} ds = \frac{V}{I} \int_0^b st \frac{h}{2} ds$$

$$= \frac{Vthb^2}{4I}$$

$$I = I_{web} + 2I_{flange} = \frac{1}{2}th^3 + 2 \left[\frac{1}{12}bt^3 \left(\frac{h}{2}\right)^2 \right]$$

$$= \frac{1}{12}th^2(6b+h)$$

* Combining

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{4 \text{ in}}{2 + \frac{6 \text{ in}}{3(4 \text{ in})}}$$

$$e = 1.6 \text{ in}$$

* Determine the Shear Stress distribution for $V = 15 + 3 = 18$ kips

$$\tau = \frac{V}{t} = \frac{VQ}{It}$$

* Shearing stresses in the flanges,

$$\tau = \frac{VQ}{It} = \frac{V}{It} (st) \frac{h}{2} = \frac{Vh}{2I}$$

$$\tau_B = \frac{Vhb}{2 \left(\frac{1}{12} th^2 \right) (6b+h)} = \frac{6Vb}{th(6b+h)}$$

$$= \frac{6(18 \text{ kips})(4 \text{ in})}{(0.15 \text{ in})(6 \text{ in})(6 \times 4 \text{ in} + 6 \text{ in})} = 8 \text{ kips/in}^2$$

* Shearing stresses in the web

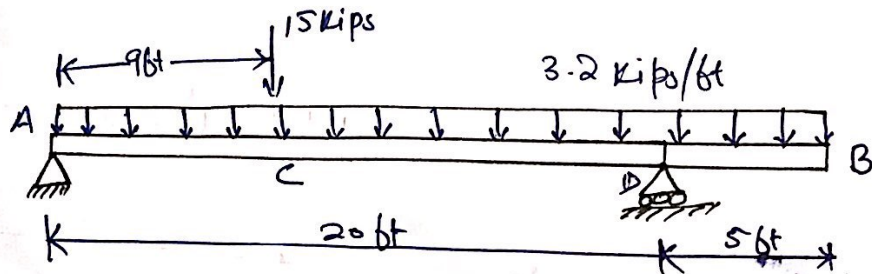
$$\tau_{\max} = \frac{VQ}{It} = \frac{V \left(\frac{1}{8} ht \right) (4b+h)}{\frac{1}{12} th^2 (6b+h) t} = \frac{3V(4b+h)}{2th(6b+h)}$$

$$= \frac{3(18 \text{ kips})(4 \times 4 \text{ in} + 6 \text{ in})}{2(0.15 \text{ in})(6 \text{ in})(6 \times 6 \text{ in} + 6 \text{ in})} = 16.6 \text{ kips/in}^2$$

Q No 5

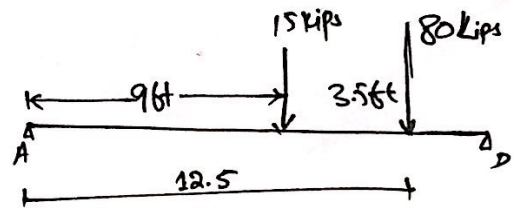
Given Data : $\sigma_{all} = 15 + 4 = 19 \text{ ksi}$

$T_{all} = 15 + 1 = 16 \text{ ksi}$

Sol.

D Reaction at A \approx D

$$= 3.2 \times 25 = 80 \text{ K}$$



Taking M at A \curvearrowright

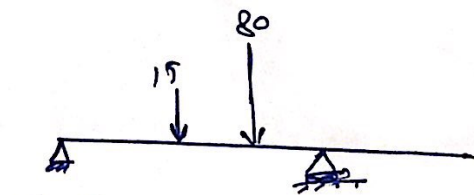
$$15 \times 9 + 80 \times 12.5 - R_D \times 20 = 0$$

$$\underline{R_D = 56.75}$$

Now M at D \curvearrowright

$$R_A \times 20 - 15 \times 11 - 80 \times 7.5 = 0$$

$$\underline{R_A = 38.25}$$

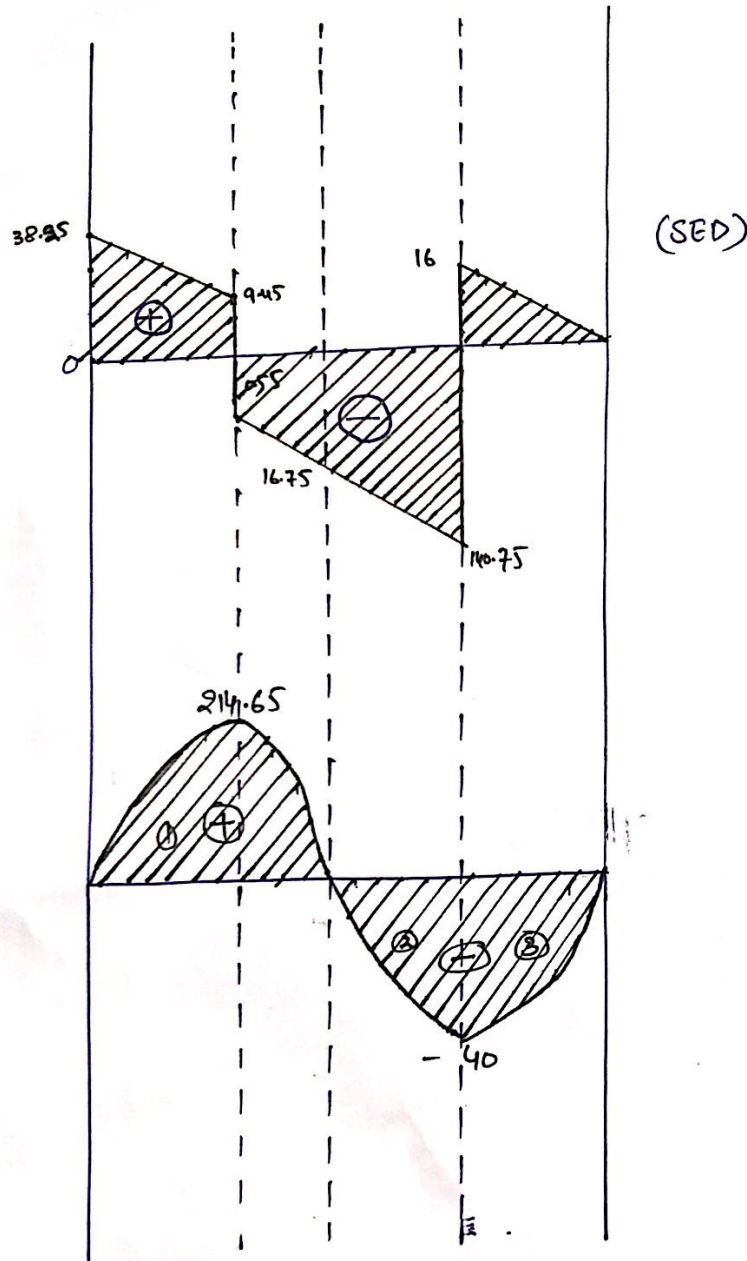
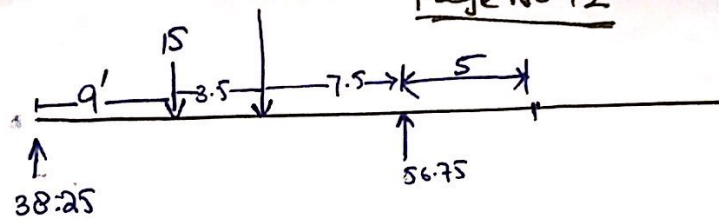


Check

$$15 + 80 = 56.75 + 38.25$$

$$95 = 95$$

2)



$$1) +ve = (38.25 - 9.45) \times (9 \times 0.5) + (9.45 \times 9) = 214.65$$

$$2) -ve = (40.75 - 55.5) \times 11 \times 0.5 + 5.5 \times 11 = 254.65$$

$$3) = (16 \times 5) \times 0.5 = 40$$

$$4) \text{ Max normal stress} = 214.6$$

$$5) \text{ Max shear} = 40.75$$