Department of Electrical Engineering
Sessional Assignment
Date: 01/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	**Sir Pir Meher Ali Shah**	**Total Marks:**	20

Student Details

Name:	**Junaid Ur Rehman**	**Student ID:**	**11484**

Q1.	(a)	Determine the response y(n), n ≥ 0, of the system described by the second order difference equation $$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$ To the input $x(n) = 4^n u(n)$.	Marks 6
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$	
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6
	(b)	Determine the partial fraction expansion of the following proper function $$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$	
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response H(ω) satisfies the condition H(0) = 1 and $\left\|H(\frac{\pi}{4})\right\|^2 = \frac{1}{2}$.	Marks 4

	(b)	Design a two-pole bandpass filter that has the center of its passband at ω = π/2, zero in its frequency response characteristics at ω = 0 and ω = π and its magnitude response in $\frac{1}{\sqrt{2}}$ at ω = 4π/9.	
Q 4	(c)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for N ≥ L	Marks 4
	(d)	Compute the DFT of the four-point sequence $$x(n) = (0\ \ 1\ \ 2\ \ 3)$$	

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Subject: Digital Signal Processing (DSP)

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NAME # JUNAID-Ur-Rehman
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Assignment Final term (dsp)

Q 1a #
SOL#

$$y_c(n) = C_1(-1)^n + C_2(4)^n$$

Normally we could assume a solution in form

$$y_p(n) = k(4)^n u(n)$$

We assume that

$$y_p(n) = kn(4)^n u(n)$$

put it in given equation

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + x(n)$$

-> By putting the equation

$$\to kn(4)^n \cancel{u} u(n) - 3k(n-1)(4)^{n-1}u(n-1) - 4k(n-2)4^{(n-2)} u(n-2)$$

$$= (4)^n u(n) + 2(4)^{n-1} u(n-1)$$

For $n=2$, $U = 6/3$

$$y_p(n) = \frac{6}{3} n (4)^n u(n).$$ difference

So the total solution to the equation

$$y(n) = C_1(-1)^n + C_2(4)^n + \frac{6}{3} n (4)^n \cdot n \geq 0$$

(2)

c_1 and c_2 are determined

$$y(0) = c_1 + c_2$$

$$y(1) = -c_1 + 4c_2 + \frac{24}{5}$$

Computing above by setting

$y(-1) = y(-2) = 0$

So

$$c_1 + c_2 = 1$$

$$-c_1 + 4c_2 + \frac{24}{9} = 9$$

$$y_{zs}(n) = \frac{-1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6n}{5}(4)^n \quad n \geq 0$$

1b Ans

Sol#

$$y(z) = \frac{X(z)}{1 - 0.6z^{-1} + 0.8z^{-2}}$$

$$X(z) = 1$$

$$\therefore H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{1}{1 - 0.6z^{-1} + 0.8z^{-1}}$$

$$= \frac{1}{(1 - 1/5\, z^{-1})(1 - \frac{2}{5} z^{-1})}$$

$$H(z) = \frac{1}{1 - 1/5\, z^{-1}} + \frac{2}{1 - 2/5\, z^{-1}}$$

$$h(n) = \left[-1 \left(\tfrac{1}{5}\right)^n + 2\left(2/5\right)^n\right] u(n)$$

(3)

Q2 A#

SOL#

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$

Hence, $x(n) = [4(2)^n - 3 - n]\,u(n)$

Q 2(b)

SOL# First we have to eleminate the negative power so we will divide and multiply z^2

$$X(z) = \frac{z^2}{z^2 - 1.5z^{+1} + 0.5z^{-2+2}}$$

$$= \frac{z^2}{z^2 - 1.5z^{-1} + 0.5}$$

$$= \frac{z^2}{z^2 - 1z^{-1} - 0.5z^{-1} + 0.5}$$

$$= \frac{z^2}{z(z-1) - 0.5(z-1)}$$

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$$X(z) = \frac{z^2}{(z-0.5)(z-1)}$$

$P_1 = 1, \; P_2 = 0.5$

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)}$$

$$\frac{z}{(z-1)(z-0.5)} = \frac{A}{(z-1)} + \frac{B}{(z-0.5)}$$ multiply $(z-1)(z-0.5)$ both sides

$$z = A(z-0.5) + B(z-1)$$

Now for $z = 1$

$$1 = A(1-0.5) + B(0)$$

$$1 = A(0.5)$$

$$\boxed{A = 2}$$

For $z = 0.5$

$$0.5 = A(z-0.5) + B(0.5 - 1)$$

$$0.5 = B(-0.5)$$

$$\boxed{B = -1}$$

put the values of A & B in equation

$$\frac{X(z)}{z} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

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Q3(a) Soln

At $W=0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence $$b_0 = (1-p)^2$$

At $W = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{\left(1-p\cos\left(\frac{\pi}{4}\right) + jp\sin\left(\frac{\pi}{4}\right)\right)^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2} + jp/\sqrt{2})^2}$$

Hence $$\frac{(1-p)^4}{\left[(1-p/\sqrt{2})^2 + p^2/2\right]^2} = \frac{1}{2}$$

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$$\sqrt{2}\,(1-p)^2 = 1+p^2-\sqrt{2}\,p$$

The value of $p = 0.32$ satisfied this equation

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$ answer.

Q3(b) Solution # the filter must have poles at

$$p_{1,2} = re^{\pm j\pi/2}$$

and zero at $z=1$ and $z=-1$. Consequently the system is

$$H(z) = G\,\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\,\frac{z^2-1}{z^2+r^2}$$

The gain factor is determine by evaluating the frequency $H(w)$ of the filter at $w = \pi/2$ Thus we have

$$H\left(\frac{\pi}{2}\right) = G\,\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

(7)

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^2+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

or equivalently

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of $r^2 = 0.7$ satisfied this equation. Therefore the system function for the desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

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Q4A

SOL#

The Fourier transform of this equation is

$$X(w) = \sum_{n=0}^{L-1} x(n) e^{-jwn}$$

$$= \sum_{n=0}^{L-1} e^{-jwn} = \frac{1-e^{-jwL}}{1-e^{-jw}} = \frac{\sin(wL/2)}{\sin(w/2)}$$

$$= \frac{\sin(wL/2)}{\sin(w/2)} \cdot e^{-jw(L-1)/2}$$

The N-point DFT of $x(n)$ is simply $x(w)$ evaluated at the set of N equally spaced frequency

$w_k = 2\pi k/N \quad k = 0, 1, 2 \ldots N-1$

Hence.

$$x(k) = \frac{1-e^{-j\pi kL/N}}{1-e^{-j2\pi k/N}} \quad k = 0, 1 \cdots N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$

Q4b #

SoL #

Fist We have to determine the matrix W_4. By exploting the periodic property of W_4 and the Symentry property

$$W_N^{k+N/2} = W_N^k$$

The matrix may be expened o

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then

$$X_4 = W_4 \, x_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$