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Subject	*Differenial Equation*
Assignment	*Final Term*
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Q No 1

$f(t) = 1+t \quad -\pi \leq t \leq \pi$

Here we use the formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \rightarrow \text{eq } 1$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2} \right) \right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{\cancel{2}} \right)$$

$$\boxed{a_0 = \frac{1}{2\pi} \left(2\pi + \pi^2 \right)}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t)(\cos nt)\, dt$$

$$a_n = \frac{1}{\pi}\left((1+t)\frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int \left(\frac{\sin nt}{n} \frac{d}{dt}(1+t) \right) \right)$$

$$a_n = \frac{1}{\pi}\left((1+t)\cancel{\frac{\sin nt}{n}}^{0} - \frac{\cos nt}{n^2} \Big|_{-\pi}^{\pi} \right)$$

$$a_n = \frac{-1}{n^2 \pi}\left(\cos n\pi - \cos n(-\pi) \right)$$

$$a_n = \frac{-1}{n^2 \pi}\left(-1 - (-1) \right)$$

$$\boxed{a_n = 0}$$

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$$bn = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t) \sin nt \, dt$$

$$bn = \frac{1}{\pi}\left((1+t) \int_{-\pi}^{\pi} \sin nt - \int\left(\left(\int \sin nt \, \frac{d}{dt}\right.\right.\right.$$

$$\left.(1+t)\, dt\right)$$

$$bn = \frac{1}{\pi}\left(-\frac{(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} - \int \frac{-\cos nt}{n}(1)\right)$$

$$bn = \frac{1}{\pi}\left(-\frac{(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \left(\cancel{\frac{\sin n\pi}{n^2}}^{\,0}\Big|_{-\pi}^{\pi}\right)\right)$$

$$bn = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - ((1+(\pi))\right.$$

$$\left.(\cos n(\pi)\right)$$

$$bn = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi \cos n\pi - \cancel{\cos n\pi} +\right.$$

$$\left.\pi \cos n\pi\right)$$

$$bn = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi} \cos n\pi\right)$$

(4)

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$b_n = \frac{2}{n}(-1)^{n+1}$

So equation become.

$$\boxed{f(x) = \frac{1}{2\pi}\left(2\pi + \pi^2\right) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nt}$$

Ans

Q NO 2

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen value

SoL:-

$\boxed{\text{Step} = 01}$

we have;

$(A - \lambda I)x = 0$ A = Given Matrix

$\boxed{\text{Step } 02}$

we have the characteristics

Equation is given by

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2)

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

$\boxed{\text{Step 03}}$

$$\lambda^3 - \left|\begin{matrix}\text{Sum of} \\ \text{Diagonal elem-}\end{matrix}\right| \lambda^2 + \left|\begin{matrix}\text{Sum of} \\ \text{Diagonal} \\ \text{minor}\end{matrix}\right| \lambda - |A| = 0 \quad \text{eq(B)}$$

Sum of Diagonal elements $= 1+1+2 = 4$

Sum of Diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By putting values in eq(B);

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \longrightarrow (C)$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

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$= 1(2-8) - 0 + 1(6-0)$

$= -6+6$

$= 0$

By puttig values in (0)

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

usig Quardatic Formula.

$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$
$b = -4$
$c = -3$

$= -(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}$

$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$

$\lambda = \frac{4+\sqrt{28}}{2}, \quad \lambda = \frac{4-\sqrt{28}}{2}$

(4)

we have eigenvalues

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$

Required solution :-

Q NO 3 Solve the following system of linear equation.

$$5x + 4 + 2m = 3$$

$$x - y + 2z + 2m = 1$$

$$4x + y + 2z = 1$$

$$x + y + z + m = 0$$

Solution:

writing in Matrix form.

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & +1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ m \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

the Augmented Matrix "Ab" is

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

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we solve the given system by Gauss Elimination method.

$$R_1 \leftrightarrow R_4 \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right]$$

$$R_2 - R_1 \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right]$$

$$R_3 - 4R_1 \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right]$$

$$R_4 - 5R_1 \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$$

$$R_2 \div (-2) \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$$

$$R_3 + 3R_2 \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & 0 & -7/2 & -4 & -1/2 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$$

$$R_4 + 5R_2 \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & 0 & -7/2 & -4 & -1/2 \\ 0 & 0 & -7/2 & -3 & 1/2 \end{array}\right]$$

$$R_3 \times -\tfrac{2}{7}\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & -7/2 & -3 & 1/2 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

Using backward substitutions

$$0\cdot x + 0\cdot y + 0\cdot z + 1\cdot m = 1$$

$$\Rightarrow \boxed{m = 1}$$

$$0\cdot x + 0\cdot y + 1\cdot z + 8/7\, m = \frac{1}{7}$$

$$z + \frac{8}{7}(1) = 1/7$$

$$z = 1/7 - \frac{8}{7}$$

$$z = \frac{1-8}{7}$$

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$$Z = \frac{-7}{7}$$

$$\boxed{Z = -1}$$

$$0 \cdot x + 1 \cdot y - \tfrac{1}{2} Z + 0 \cdot m = -\tfrac{1}{2}$$

$$y - \tfrac{1}{2} Z = -\tfrac{1}{2}$$

$$y - \tfrac{1}{2}(-1) = -\tfrac{1}{2}$$

$$y + \tfrac{1}{2} = -\tfrac{1}{2}$$

$$y = -\tfrac{1}{2} - \tfrac{1}{2}$$

$$y = \frac{-1-1}{2}$$

$$y = \frac{-2}{2}$$

$$\boxed{y = -1}$$

(6)

$$1 \cdot x + 1 \cdot y + 1 \cdot z + 1 \cdot m = 0$$

$$x + (-1) + (-1) + 1 = 0$$

$$x - 1 - \cancel{1} + \cancel{1} = 0$$

$$\boxed{x = 1}$$

So

$x = 1$, $y = -1$ $z = -1$ & $m = 1$

is the Solution.

x

①

Q4:
$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(x,t) = \sin(x+2t) \quad \text{is solution of}$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

if will ~~sta~~ satisfy the above equations.

$$\frac{\partial u}{\partial t} = \cos(x+2t) \cdot \frac{d}{dt}(x+2t)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

again.

$$\frac{\partial^2 u}{\partial t^2} = -2\sin(x+2t) \cdot \frac{d}{dt}(x+2t)$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)} \rightarrow Ⓐ$$

Now $\frac{\partial u}{\partial x} = \cos(x+2t)$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)} \rightarrow (B)$$

Comparing (A) & (B)

$c = 2.$

$\Rightarrow \quad -4\sin(x+2t) = -c^2 \sin(x+2t)$

$\Rightarrow \quad -4\sin(x+2t) + c^2 \sin(x+2t) = 0$

This is possible if $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$0 = 0$$

Thus $u(x,t) = \sin(x+2t)$ is solution of 1-D wave equation.