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Subject : Differential equation

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Section : Summer Semister

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Question NO. (01)

(i) Solve the intial value problem

$$dy/dx = e^{y-t}\sec(y)(1+t^2) \quad y(0)=0$$

Solution:

$$= \frac{dy}{dx} = \frac{e^{y}}{e^{t}} \times \frac{1}{\cos y}(1+t^2)$$

$$\text{or} = e^{-y}\cos y\,dy = e^{-t}(1+t^2)\,dt$$

Integrating = Both sides

$$= \int e^{-y}\cos y\,dy = \int e^{-t}(1+t^2)\,dt \text{ --- (i)}$$

By parts

$$= \cos y\,\frac{e^{-y}}{-1} - \int -\sin y\,\frac{e^{-y}}{-1} + \int \cos y.\,e^{-y}dy$$

$$= \int e^{-t}(1+t^2)\,dt$$

$$= -e^{-y}.\cos y - [-\sin y\,e^{-y} + \int \cos y\,e^{-y}dy = \int e^{-t}(1+t^2)\,dt$$

$$= -e^{-y}\cos y + e^{-y}\sin y - \int \cos y\,e^{-y}dy = \int e^{-t}(1+t^2)\,dt$$

$$= e^{-y}(\sin y - \cos y) - \int \cos y.\,e^{-y}dy = \int e^{-t}(1+t^2)\,dt$$

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From equation No. (i)

$= \int e^{-t}(1+t^2)dt = \int e^{-y}\cos dy$

So

$= e^{-y}(\sin y - \cos y) - \int e^{-t}(1+t^2)dt -$
$= \int e^{-t}(1+t^2)dt$

$= e^{-y}(\sin y - \cos y) = \int e^{-t}(1+t^2)dt + \int e^{-t}(1+t^2)dt$

$= e^{-y}(\sin y - \cos y) = 2\int e^{-t}(1+t^2)dt$

$= e^{-y}(\sin y - \cos y) = 2(1+t^2)\frac{e^{-t}}{-1} + \int 2t.e^{-t}dt$

$= e^{-y}(\sin y - \cos y) = 2[e^{-t}(1+t^2) + 2(t\frac{e^{-t}}{-1} + \int e^{-t}dt)]$

$= e^{-y}(\sin y - \cos y) = 2[e^{-t}(1+t^2) + 2[t.e^{-t} - e^{-t}] + C$

$= e^{-y}(\sin y - \cos y) = 2[-e^{-t} - e^{-t}.t^2 - 2te^{-t} - 2e^{-t}] + C$

$= e^{-y}(\sin y - \cos y) = 2[-3e^{-t} - t^2e^{-t} - 2te^{-t}] + C$

$= e^{-y}(\sin y - \cos y) = -2e^{-t}[t^2 + 2t + 3] + C$

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Question No. (02)

Solution:

$$\left(\sqrt{x+y}+\sqrt{x-y}\right)dx-\left(\sqrt{x+y}-\sqrt{x-y}\right)dy=0$$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}} \Rightarrow (1)$$

This is Homogeneous Differential equation in x and y to solve this put $y=vx$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Thus the equation (1) becomes

$$= v+x\frac{dv}{dx}=\frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$= v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$= v + x\frac{dv}{dx} = \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}} \times \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}+\sqrt{1-v}}$$

$$= v + x\frac{dv}{dx} = \frac{1+v+1-v+2\sqrt{1-v^2}}{2v}$$

$$= v + x\frac{dv}{dx} = \frac{\cancel{2}(1+\sqrt{1-v^2})}{\cancel{2}v}$$

$$= v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$= x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$= x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

$$= x\frac{dv}{dx} = \frac{\sqrt{1-v^2}(1+\sqrt{1-v^2})}{v}$$

$$= \frac{v\,dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2}} = \frac{dx}{x}$$

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Taking integrate on both sides.

$$\int \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

Put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \tfrac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$= \frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$= \int \frac{-dt}{t} = \int \frac{dx}{x}$$

$$= -\ln t = \ln x + \ln c$$

$$= -\ln(1+\sqrt{1-v^2}) = \ln cx$$

$$\ln(1+\sqrt{1-v^2}) = \left(\frac{1}{cx}\right)\ln$$

$$= 1+\sqrt{1-v^2} = 1/cx$$

$$= 1+\sqrt{\frac{x^2-y^2}{x^2}} = 1/cx$$

$$= x+\sqrt{x^2-y^2} = 1/c$$

$$\because \frac{1}{c} = c_1$$

$$x+\sqrt{x^2-y^2} = c_1$$

This is required solution.

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Question No. (03)

$$(D^4+D^2)y = 3x^2+4\sin x-2\cos x$$

Solution:

$$=(D^4+D^2)y = 3x^2+4\sin x-2\cos x$$

$$\Rightarrow f(D)y = f(x)$$

As it is non-homogenuous linear equation so solution will be:

$$y = y_c+y_p \rightarrow (i)$$

complementry solution y_c

$$D^4+D^2=0 \Rightarrow D^2(D^2+1)=0$$

Either $D^2=0 \Rightarrow \boxed{D=0}$

$$D^2+1=0 \Rightarrow D^2 = 1$$

$$D=\sqrt{-1} \Rightarrow \boxed{D=i} \text{ or } \boxed{D=0+i}$$

Roots are real and complex

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$$y_c = c_1 e^{0x} + e^{0x}(c_2 \cos x + c_3 \sin x)$$

$$y_c = c_1 + c_2 \cos x + c_3 \sin x$$

$$= \quad y_p = \frac{1}{f(D)} F(x)$$

$$= y_p = \frac{1}{D^4 + D^2}(3x^2 + 4\sin x - 2\cos x)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$$f(D) = D^4 + D^2$$

at $D=0 \Rightarrow \quad f(D) = 0$

SO $\quad f'(D) = 4D^3 + 2D$

Now also for $D=0 \Rightarrow f'(D) = 0$

Again Differentiating.

$$f''(D) = 12D + 2$$

So for $D=0$

$$f''(0) = 12(0) + 2 = 2$$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

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$$\Rightarrow y_p = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} \cdot 2\cos x$$

putting $D=0$ in all

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

so

putting in equation (i)

$$y = c_1 + c_2 \cos x + c_3 \cos x + \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

$$\boxed{y = c_1 + (c_2 - x^2)\cos x + (c_3 + 2x^2)\sin x + \frac{3}{2}x^4}$$

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