Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: ____________ **Total Marks:** 50

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Student Details

Name: ____________ **Student ID:** ____________

Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks** 7 / **CLO** 2
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	**Marks** 7 / **CLO** 2
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	**Marks** 6 / **CLO** 2
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z\rvert > \lvert a\rvert$$	**Marks** 6 / **CLO** 2
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$ Determine the values of b_0 and p such that the frequency response $H(\omega)$ satisfies the condition H(0) = 1 and $\left\lvert H\left(\frac{\pi}{4}\right)\right\rvert^2 = \frac{1}{2}$.	**Marks** 6 / **CLO** 3

Q_1 (c)

solution

$$y(n)-4y(n-1)+4y(n-2)=x(n)-x(n-1)$$

The characteristic equation is

$$\lambda^2-4\lambda+4=0$$

$\lambda = 2,2$ Hence

$$y_h(n)=C_1 2^n+C_2 n 2^n$$

The particular solution is

$$y_p(n)=K(-1)^n u(n)$$

substituting this solution into the the different equation, we obtain.

$$K(-1)^n u(n)-4K(-1)^{n-1}u(n-1)+4K(-1)^{n-2}u(n-2)=(-1)^n u(n)-(-1)^{n-1}u(n-1)$$

For $n=2$, $K(1+4+4)=2$

$K=\frac{2}{9}$. The Total solution is

$$y(n)=\left[C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n\right]u(n)$$

From the initial condition,

we obtain ~~y(0)=1~~ $y(0)=1$,

$y(1)=2$ then

$$C_1+\frac{2}{9}=1$$

$$C_1=\frac{7}{9}$$

$$2C_1+2C_2-\frac{2}{9}=2$$

$$C_2=\frac{1}{3}$$ 2

So put in Equation (i)

$y(n)\left[\frac{7}{2}2^n+\frac{1}{3}n\,2^n+\frac{2}{9}(-1)^n\right]u(n)$

Page#2

Ans

Q1(b)

$$y(n) - 0.7y(n-1) + 0.1y(n-2) -$$
$$2x(n) - x(n-2)$$

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2} \cdot \frac{1}{5}$ Hence

$$y_n(n) = C_1 \frac{1^n}{2} + C_2 \frac{1^n}{5}$$

$$y(0) = 2$$

$$y(1) - 0.7y(0) = 0$$

$$y(1) = 1.4$$

Hence equation yield

$$C_1 = \frac{10}{3}, \quad C_2 = -\frac{4}{3}$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u(n)$$

The step response is:

$$s(n) = \sum_{k=0}^{n} h(n-k),$$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{8}\right)^{n-k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k} - \frac{4}{3} \left(\frac{1}{8}\right)^{n} \sum_{k=0}^{n} 8^{k}$$

$$= \frac{10}{3} \left(\frac{1}{2}^{n} \left(2^{n+1} - 1\right) u(n) - \frac{1}{3} \left(\frac{1}{8}^{n} \left(8^{n+1} - 1\right) u(n) \right.\right.$$

Ans

Page # 4

2(ci)

Ans

Solution.

Determine the causal signal ~~x(n)~~ $x(n)$ having the z-transform.

$$X(z) = \frac{1}{(1+2z^{-1})(1-z^{-1})^2}$$

$$X(z) = \frac{1}{4(1+2z^{-1})} + \frac{3}{4}\frac{1}{1-2z^{-1}} + \frac{1}{2}\frac{z^{-1}}{(1-z^{-1})^2}$$

By Applying inverse transform

$$x(n) = \frac{1}{8}(-1)^n u(n) - \frac{3}{8}u(n) + \frac{1}{2}nu(n)$$

$$= \left[\frac{1}{8}(-1)^n + \frac{3}{8} + \frac{n}{2}\right]u(n) \text{ Ans}$$

Q2(b) $X(z) = \frac{1}{1-az^{-1}} \quad |z| > |a|$

solution.

we have

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}} dz = \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z-a}$$

where c is a circuile at radius greater than than $|a|$. we shall evaluate the integral using '(3.4.2) with $f(z) = z^n$ we distinguish Two Cases.

The z-Transform and its Application to the Analysis of LTI system.

1. if $n \geq 0$, $f(z)$ has only zeros and hence no poles inside C. The only pole inside C is $z=a$ Hence.

$$x(n) = f(z_0) = a^n \quad n \geq 0$$

P.T.O

2. If $n<0$, $f(z)=a z^n$ has an nth-order pole at $z=0$ which is also inside C. Thus there are contributions from both poles.

for $n=-1$ we have

$$x(-1)=\frac{1}{2\pi j}\oint_C \frac{1}{z(z-a)}dz=\frac{1}{z-a}\bigg|_{z=0}+\frac{1}{z}\bigg|_{z=a}=0$$

If $n=-2$ we have

$$x(-2)=\frac{1}{2\pi j}\oint_C \frac{1}{z^2(z-a)}dz=\frac{d}{dz}\left(\frac{1}{z-a}\right)\bigg|_{z=0}+\frac{1}{z^2}\bigg|_{z=a}=0$$

By continuing in the same way we can show that $x(n)=0$ for $n<0$ Thus

$x(n)=a^n u(n)$ Ans

2

Q.3(a)

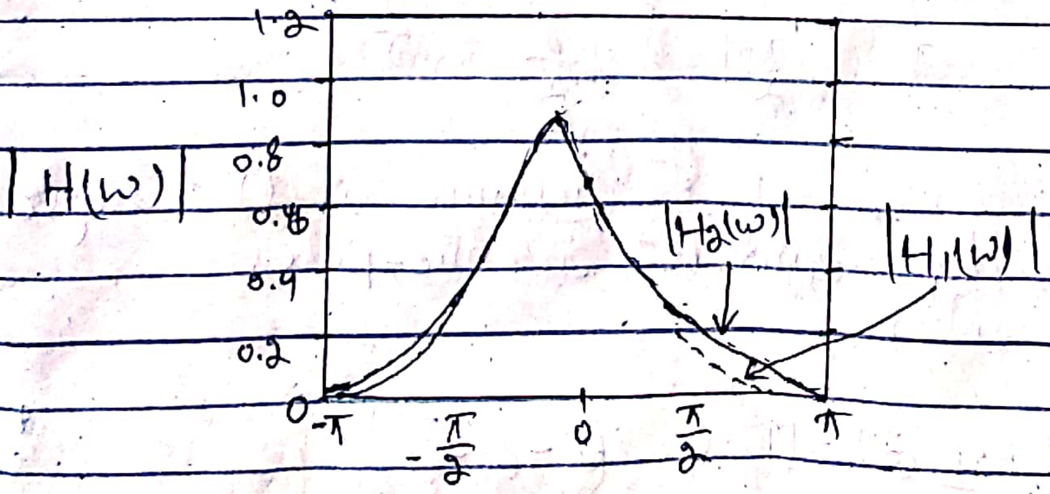
A Two-pole low pass filter has the system response.

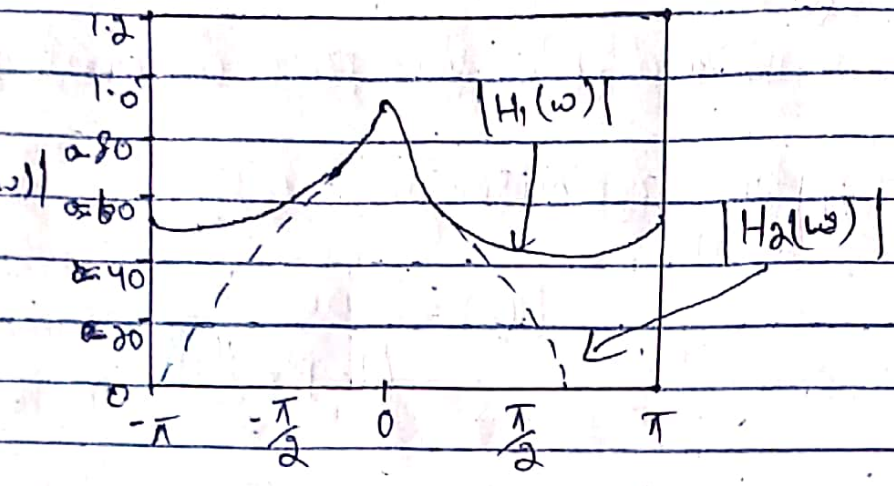
$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

Ans.

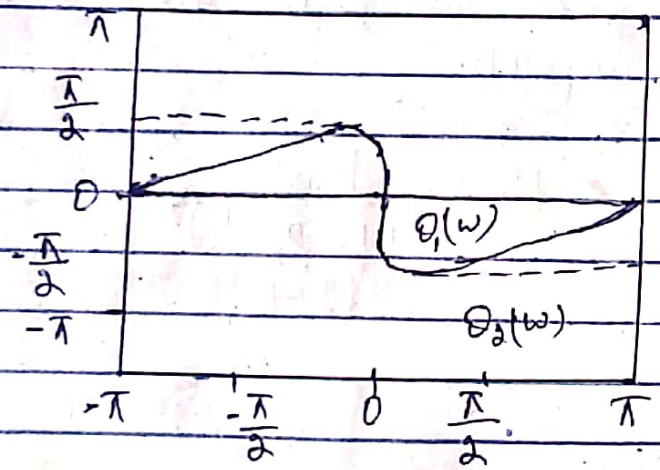
A Two pole low pass filter has the system function.

$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$





$\Theta(\omega)$



Magnitude and phase response of (1) a single pole filter and (2) a one-pole, one zero filter.

$$H_1(z) = (1-a) / (1-az^{-1})$$

$$H_2(z) = [(1-a)/2] [(1+z^{-1}) / 1-az^{-1})]$$

and $a = 0.9$.

Determine the value of b_0 and p Such the Frequency response $H(\omega)$ satisfies the Condition.

$$H(0) = 1$$

and

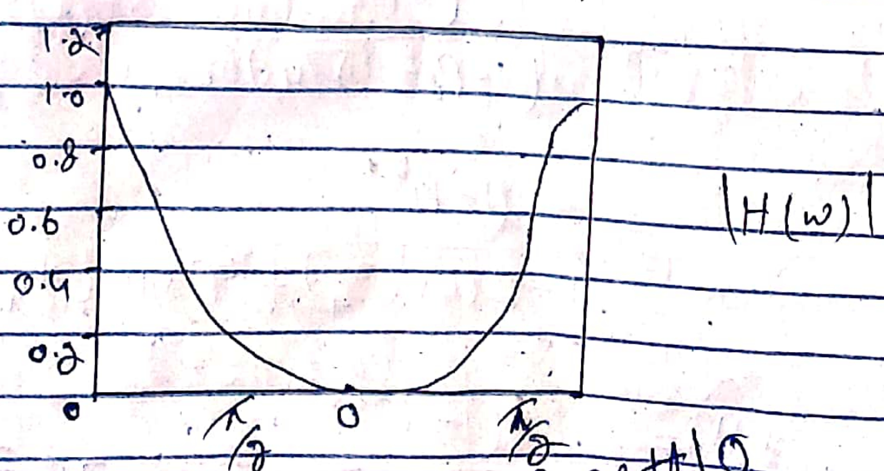
$$\left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{1}{2}$$

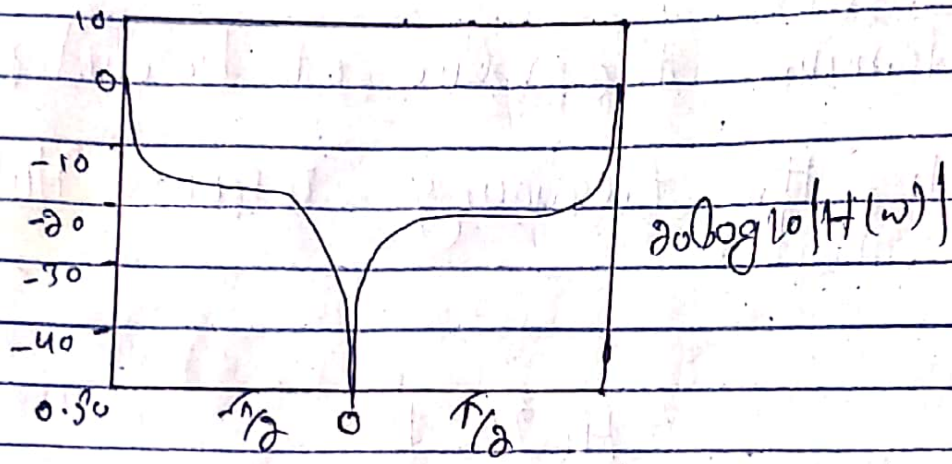
<u>solution</u>

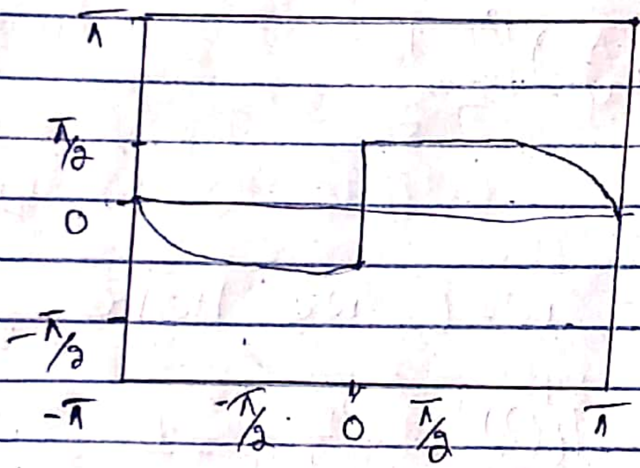
At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence $b_0 = (1-p)^2$







At $w = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{1-P\cos(\pi/4)+jP\sin(\pi/4)^2}$$

Page#11 $$= \frac{(1-P)^2}{[(1-P/\sqrt{2})^2+jP/\sqrt{2}]^2} = \frac{1}{2} \quad 4$$

or equivalently

$$\sqrt{2}(1-p)^2 = 1 + p^2 - \sqrt{2}p$$

$p = 0.32$

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$ Ans.

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Q3b.

Solution $p_2 = re^{\pi/2}$

and zero at z=1 and z=-1,

Consequently the system function is

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{z^2-1}{z^2-r^2}$$

The gain factor is Determine by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$ we here

$$H\left(\frac{\pi}{2}\right) = G \frac{2}{1-r^2} = 1$$

The value of r is determine be evaluating $H(\omega)$ at $\omega = 4\pi/9$ Thus we here

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

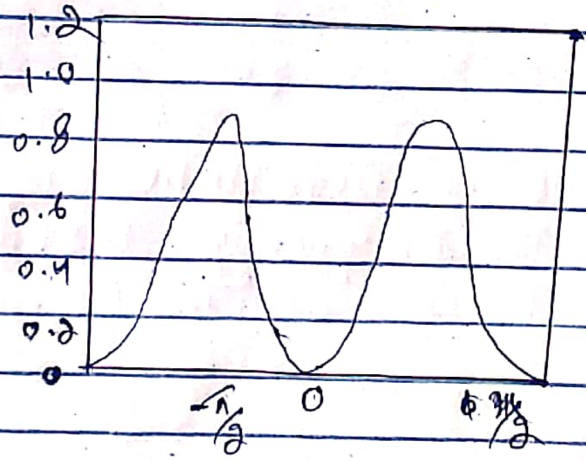
or. equivalently.

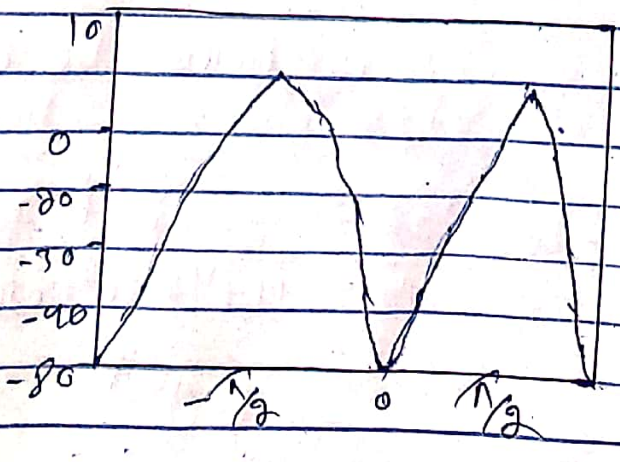
$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of $r^2 = 0.7$ satisfies the Equation.

There fore the system Function For the desired filter is.

$$H(z) = 0.15\,\frac{1-z^{-2}}{1+0.7z^{-2}}$$





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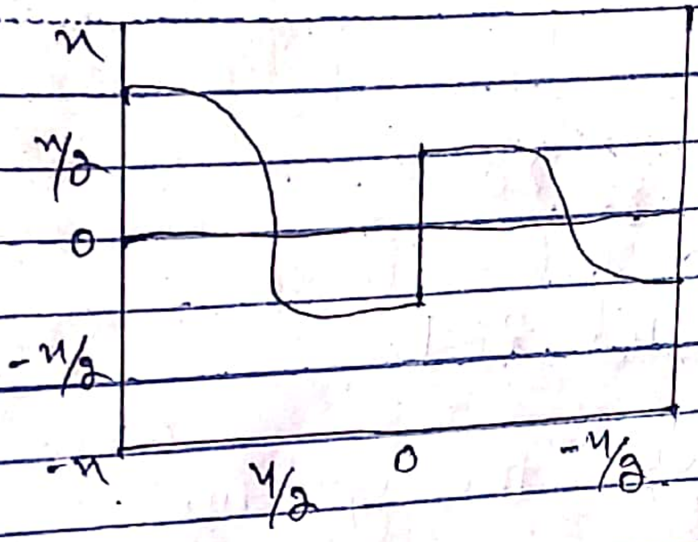

n
n/2
0
-n/2
-n
n/2
0
-n/2

Q4(a) $x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0 & \text{otherwise} \end{cases}$

Determine the N-point DFT of this sequence of $N \ge L$.

Solution:

The Fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

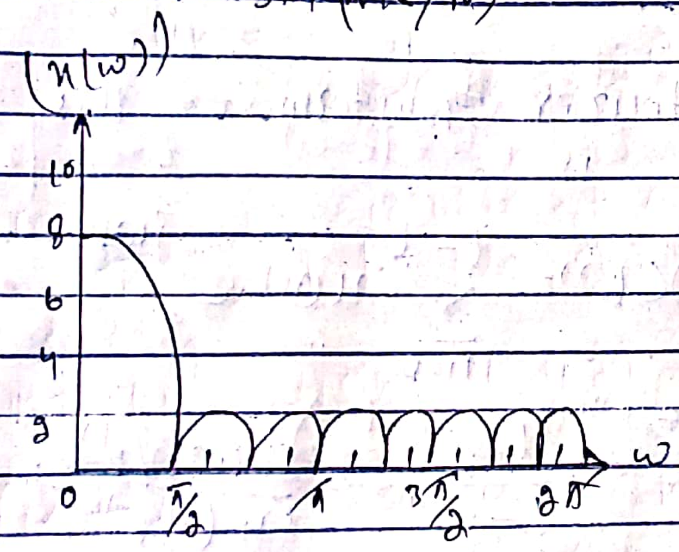
$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

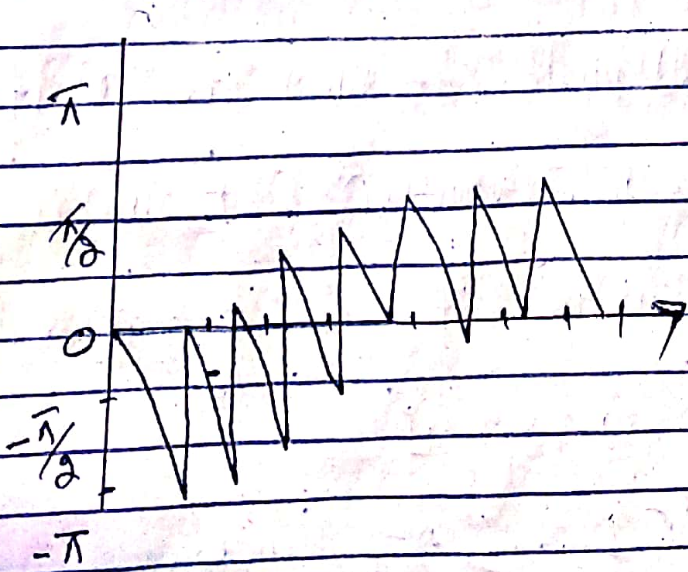
The magnitude and phase of $X(\omega)$ are illustrated in fig 8.8 for $L=10$ The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally spaced frequencies

$\omega_k = 2\pi K/N$, $K = 0, 1 \ldots\ldots N-1$ Hence.

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}} \qquad K = 0, 1, \dots, N-1$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





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If N is Selected such that N=L then the

DFt becomes.

$$x(k) = \begin{cases} L & k=0 \\ 0 & k=1,2 \cdots L-1 \end{cases}$$

Thus There is one nonzero value in DFt is opparant from observation of $X(\omega)$ since $X(\omega)=0$ at the frequency.

$\omega_k = 2\pi k/L$ $k \neq 0$ The reader should verify hat $x(n)$ can be recovered From $X(k)$ by performing L-Point IDFT.

Q4 b:

Ans

$$x_1(n) = \{2, 1, 2, 1\}$$
$$x_2(n) = \{1, 2, 3, 4\}$$

Each sequence Consist of Four nonzero points.
For the purposes of illustrating the operation ~~included~~ involded in Circular Convolution.

Now $x_3(m)$ is obtained by Circularly Convolving $x_1(n)$ with $x_2(n)$ Begining with $m=0$

we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n)(x_2(-n))_N$$

$x_2(1-n)_4$ is simply the sequence $x_2(n)$ Folded and graph on a

1

Circle as illustrated in the graph

$$x_3(0) = 14$$

For $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

It is easily verified that

$x_2((1-n))_4$ is simply the sequence

$x_2((-n))_4$ rotated counter clock wise.

$$x_3(1) = 16$$

For $m=2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, X_2\,(2-n)_4$$

Now $x_2(2-n)_4$ is the Folded 2

Sequence.

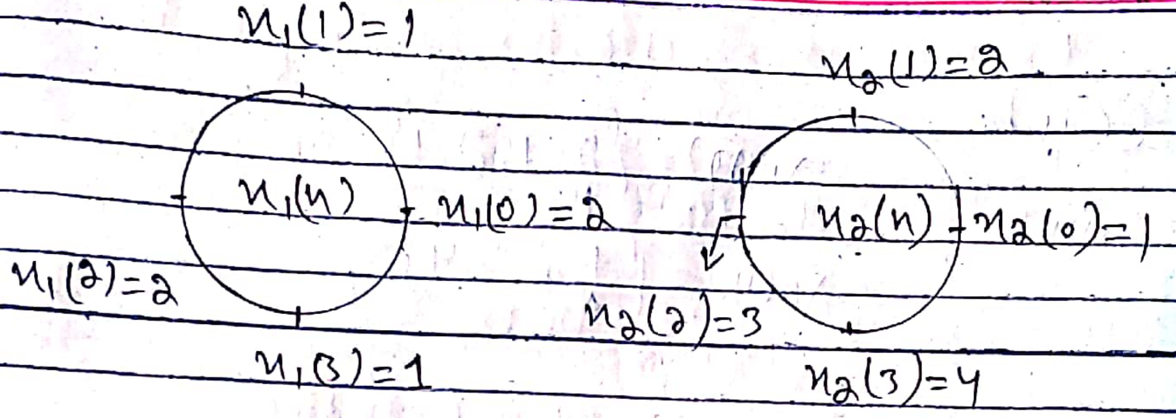
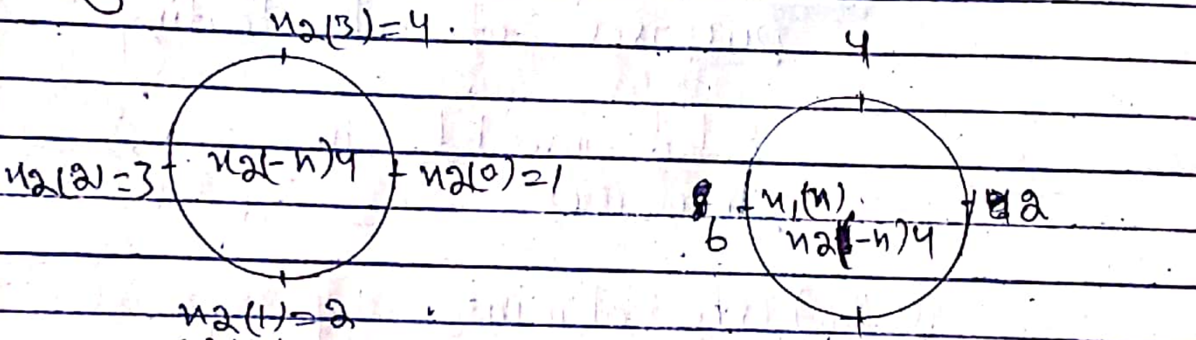


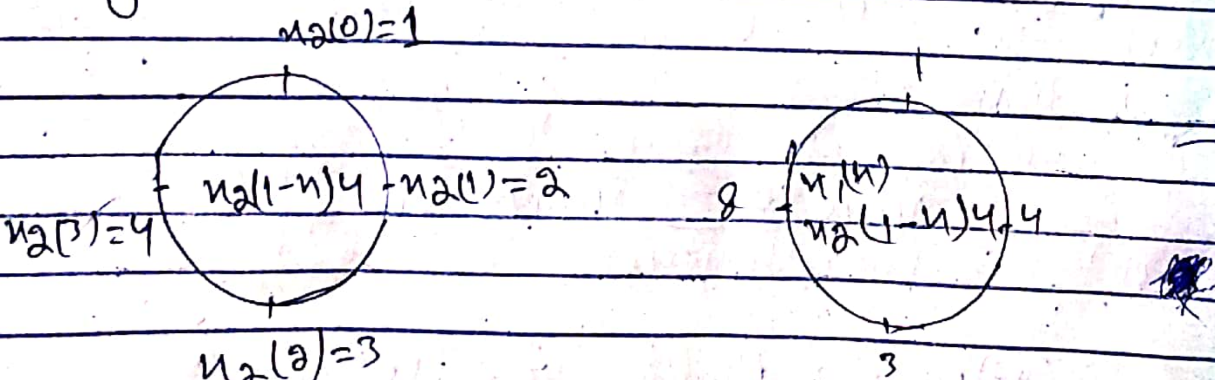
Figure (a)



Folded sequence

Product sequence.

Figure (b)

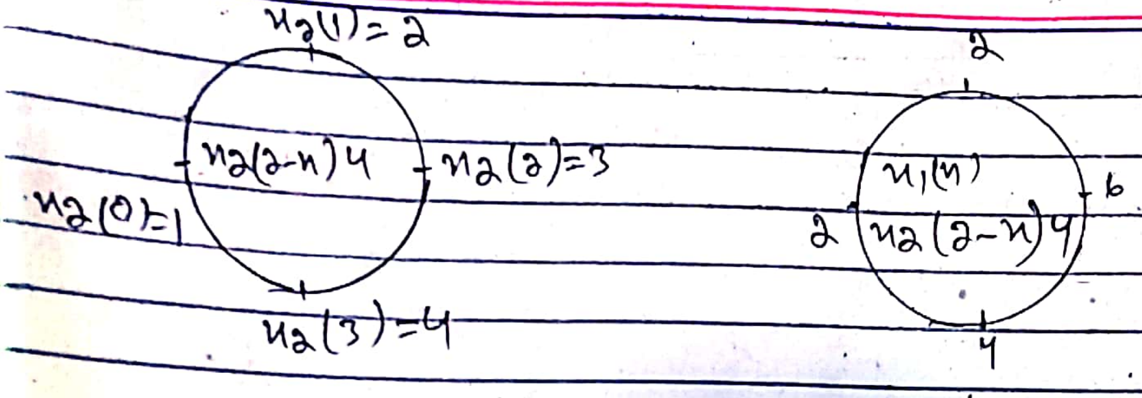


Folded sequence rotated by one unit in time.

Product sequence

Page #

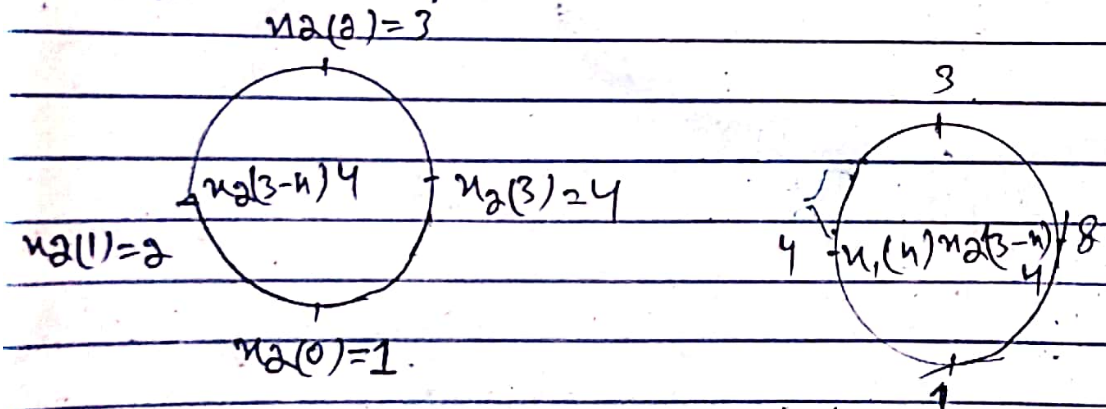
Figure (C)



Product sequence

Folded sequence rotated by Two units in Time.

figure (d)



Folded sequence rotated by Three units in Time.

figure (e)

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long with product sequence $x_1(n)\, x_2(2-n)_4$

By the ~~o~~ summing the four Terms

in the product sequence.

$$X_3(2) = 14$$

$m = 3$ we have

$$X_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$

The folded sequence $x_2((n))_4$ is

now three unit to yield

$$X_2((3-n))_4 \text{ is}$$

multiplied $x_1(n)$ to yield the

product.

Sequence.

$$X_3 = (3)\,16$$

we observe that if the Computation.

beyond $m=3$

sequencies $x_1(n)$, $x_2(n)$)

$$x_3(n) = \{14, 16, 14, 16\}$$