SNO-1

"DIFFERENTIAL EQUATION"

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Question # 1

(a) Define differential equation along with two examples.

*** Differential Equation:** It is an equation with function/functions and its one or more derivatives

• **Examples**

(i) $y' = x^2 - 3$

$$\frac{dy}{dx} = (x^2 - 3)$$

$$dy = (x^2 - 3)dx$$

* Taking "$\int$" on B.S

$$\int dy = \int x^2 dx - \int 3\, dx$$

$$y = \frac{x^3}{3} - 3x + k$$

(k is constant)

(ii) $\frac{dy}{dx} = 6$

$x = 0$, $y = 2$

$$dy = 6\, dx$$

* Taking "$\int$" on B.S

$$\int dy = \int 6\, dx$$

$$y = 6x + k$$

∴ By applying condition

$k = 2$
$x = 0$
$y = 2$

$$y = 6x + 2$$

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(b) Define a seperatable differential equation.

If any differential equation that we can write in the following form are seperatable differential equations

$$N(y)\frac{dy}{dx} = M(x)$$

i) Solve the following IVP using SDE & find the interval of ~~valadit~~ valadity of the solution.

$$y' = \frac{xy^3}{\sqrt{1+x^2}} \qquad y(0) = -1$$

* Solution :-

$$y' = \frac{xy^3}{\sqrt{1+x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$dy \cdot y^{-3} = x(1+x^2)^{\frac{1}{2}}\, dx$$

* Integrate on B.S

$$\int y^{-3}\, dy = \int x(1+x^2)^{\frac{1}{2}}\, dx$$

$$\frac{y^{-2}}{2} = \sqrt{1+x} + C$$

$$-\frac{1}{2y^{+2}} = \sqrt{1+x} + C \quad \text{--- (1)}$$

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* putting $x = 0$ and $y = -1$

$$\frac{-1}{2^{(-1)^2}} = \sqrt{1 + (0)} + C$$

$$\Rightarrow \frac{-1}{2} = \sqrt{1} + C$$

$$C = \frac{-3}{2}$$

* Equation (i) Becomes

$$\frac{1}{-2y} = \sqrt{1+x} - \frac{3}{2}$$

* By multiplying 2 on B.S

$$\frac{1}{y} = 3 - 2\sqrt{1+x^2}$$

* Interval of valadity

$$3 - 2\sqrt{1+x^2} > 0$$

$$\Rightarrow 3 > 2\sqrt{1+x^2}$$

* Taking Square Root on B.S

$$9 > 4(1+x^2)$$

$$9 > 4 + 4x^2$$

$$9 - 4 > 4x^2$$

$$\frac{5}{4} > \frac{4}{4}x^2$$

$$\frac{5}{4} > x^2$$

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Taking square Root on B.S

$$\sqrt{\frac{5}{4}} > \sqrt{x^2}$$

$$\Rightarrow -\sqrt{\frac{5}{4}} < x < \sqrt{\frac{5}{4}}$$

* finding value of constant "C"

$x = 5$ and $y = 0$

$$0 = \ln[(5)^2 - 4(5) + C]$$

$$0 = \ln(5 + C)$$

$$5 + C = 1$$

$$C = 5 - 1$$

$$C = 4$$

* After value of C

$$y = \ln(x^2 - 4x - 4)$$

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Question No 2:
Solve the following IVP using differential equation.
Linear

Here are the steps for solving method.
linear differential equation

(i) Step 1) Substitute $y = uv$ and

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

into

$$\frac{dy}{dx} = P(x)y = Q(x)$$

Step 2) Factor the parts including v.

Step 3) Put the v term equal to zero

Step 4) Solve using seperable differential equation to find u.

Step 5) Substitute U back into equation.

Step 6) Solve that to find V

Step 7) finally substitute U and V into $y = uv$ to get our Solution.

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(ii) $\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x)$
-1 , $\therefore y\left[\frac{\pi}{4}\right] = 3\sqrt{2}$, $0 \le x \le \frac{\pi}{2}$

* Solution:

$y' + \frac{\sin(x)y}{\cos(x)} = 2\cos^2(x)\sin(x) - \frac{1}{\cos x}$

$y' + \tan(x)y = 2\cos^2(x)\sin(x) - \sec x$

$U = e^{\int \tan(x)dx} = e^{\ln(\sec(x))} = e^{\ln \sec x}$

$\Rightarrow \sec(x)y^3 + \sec(x)\tan(x)y = 2\sec(x)$

$\Rightarrow \cos^2 x \sin(x) - \sec^2(x)$

$\Rightarrow [(\sec(x)y)]^2 = 2\cos(x)\sin(x) - \sec^2(x)$

$\Rightarrow \int (\sec(x)\, y(x))' \, dx = \int 2\cos(x)\sin(x) - \sec^2(x)$

$\Rightarrow \sec x \, y(x) = \frac{-1}{2}\cos 2x - \tan x + C$

$\Rightarrow y(x) = \frac{-1}{2}(\cos x)(\cos 2x) - \cos x \tan x + \cos x$

$= y(x) = \frac{-1}{2}\cos x \cos 2x - \sin x + \cos x$

* putting $y = 3\sqrt{2}$ & $x = \pi/4$

$3\sqrt{2} = y\left(\frac{\pi}{4}\right) = \frac{-1}{2}\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{4}\right)$

$3\sqrt{2} = \frac{\sqrt{2}}{2} + C\frac{\sqrt{2}}{2}$

$C = 7$

It becomes $\Rightarrow y(x) = \frac{-1}{2}\cos(x)\cos(2x) - \sin x + 7\cos x)$

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Question NO 3:

(i) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0$, $\therefore y(0) = -3$

Solution.

$M = 2xy - 9x^2$ $\qquad M_y = 2x$

$N = 2y + x^2$ $\qquad N_x = 2x$

* Finding $\psi(x, y)$

$\psi_x = M$

$\psi_y = N$

$\psi = \int M dx$ or $\psi = \int N dy$

$\psi_y = x^2 + h'(y) = 2y + x^2 + 1 = N$

$h'(y) = 2y + 1$

$h(y) = \int 2(y) + 1 dy = y^2 + y + k$

$\Rightarrow y^2 + (x^2 + 1)y - 3x^2 + k = C$

$\Rightarrow y^2 + (x^2 + 1)y - 3x^2 = C - K$

$y^2 + (x^2 + 1)y - 3x^2 = C$

* Putting value of x and y

$(-3)^2 + (0 + 1)(-3) - 3(0)^2 = C$

$C = 6$

$y^2 = (x^2 + 1)y - 3x^2 - 6 = C$

* Quadratic Formula.

$$y(x) = \frac{-(x^2+1) \pm \sqrt{(x^2+1)^2 - 4(1)(-3x^2 - 6)}}{2(1)}$$

$$= -\frac{(x^2 + 1) \pm \sqrt{x^2 + 12x\,3 + 2x^2 + 25}}{2}$$

$y(x) = -(x^2 + 1) - \sqrt{x^4 + 12x^2 + 25}$

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(ii) $\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2-1))y' = 0$

$\therefore y(5) = 0$

* Solution:

$M = \frac{2ty}{t^2+1} - 2t$

$N = \ln(t^2+1) - 2$

$M_y = \frac{2t}{t^2+1}$

$N_t = \frac{2t}{t^2+1}$

* $\int$ of "M"

$\psi(x,y) = \int \frac{2ty}{t^2+1} - 2t \; dy = y\ln(t^2+1) - t + h(y)$

* Differentiate.

$\psi_y = \ln(t^2+1).\ h'(y)\ \ln(t^2+1) - 2$

$h'(y) - 2 \Rightarrow h(y) = -2y$

$\psi\ \ln(t^2+1) - t^2 - 2y = C$

* By putting value we get C

$y\ln(t^2+1) - 2y - t^2 = 25$

$y(t) = \frac{t^2 - 25}{\ln(t^2+1) - 2}$

$\ln(t^2+1) - 2 = 0$

$\ln(t^2+1) = 2$

$t^2 + 1 = e^2$

$t = \pm\sqrt{e^2 - 1}$