AWP
M T W T F S

Date: ..........

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Date = 19 Aug 2020

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Question no 1 a Part

Find the Polynomials of degree 3 or less that interpolates the point (0,2), (1,1), (2,0) & (3,-1)

According to given condition:

The Lagrange form is

$$P(x) = 2\frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)} + 1\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)} +$$

$$0\frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)} - 1\frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}$$

$$\Rightarrow -\frac{1}{3}(x^3-6x^2+11x-6) + \frac{1}{2}(x^3-5x^2+6x) - \frac{1}{6}$$

$$(x^3-3x^2+2x)$$

$$\Rightarrow -\frac{1}{3}(x^3-5x-6) + \frac{1}{2}(x^3-x)\ \frac{1}{6}(x^3-x)$$

$$\Rightarrow \boxed{-x+2} \rightarrow \text{Answer.}$$

Hence Proved

x — x — x

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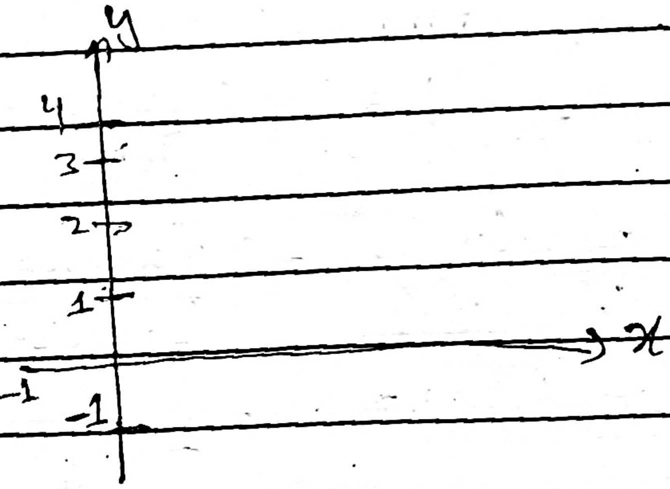
## Question no 1 B Part

Find the interporating polymomial for the data Points.

(0, 1), (2, 2) & (3, 4) fore the fig given below




Subsituting into Lagrange formula yields

$$P_2(x) = 1\frac{(x-2)(-3)}{(0-2)(0-3)} + 2\frac{(x-0)(x-3)}{(2-0)(2-3)} + 4\frac{(x-0)(x-2)}{(3-0)(3-2)}$$

$$\Rightarrow \frac{1}{6}(x^2-5x+6) + 2\left(-\frac{1}{2}\right)(x^2-3x) + 4\left(\frac{1}{3}\right)(x^2-2x)$$

$$\Rightarrow \frac{1}{2}x^2 - \frac{1}{2}x + 1$$

So $P_2(0) = 1$

$P_2(3) = 4$

for generally Purpose n point $(x_i, y_i)$ for the degree polymnol n-1

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$$Lk(x) = \frac{(x-x_1)}{(xk-x_1)} \cdots \frac{(x-xk-1)}{(xk-xk-1)} \frac{(x-xk+1)}{(xk-xk+1)} \cdots \frac{(x-xn)}{(xk-xn)}$$

The Poperly of $l_k$ & $Lk(kx) = 1$
while $Lk(xj) = 0$

$$Pn-1(x) = y_1 L_1(x) + \cdots yn\,Ln(x)$$

subsituting $xk$ for $x$ yields

$$\boxed{\begin{aligned} Pn-1(xk) &= y_1 L_1(xk) + \cdots + yn\,Ln(xk) = 0 \\ &+ \cdots + 0 + ykLk(xk) + 0 + 0 \cdots + 0 = Yk \end{aligned}}$$

Hence Proved

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Question no 2(a):

Use the two Piont forward difference formula with $h = 0.1$ approximate the derivative of $(x) = \frac{1}{x}$ at $x = 2$

Solution =

As we know that

The two Point forward difference formula

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h}$$

$$f'(x) \simeq \frac{1/2.1 - \frac{1}{2}}{0.1}$$

$$\boxed{f(x) = -0.2381} \longrightarrow ①$$

The differnce b/w this appriximation and the correct derivative

$f'(x) = -x^{-2}$ at $x = 2$ is the error

$-0.2381 - (-0.2500)$

$$\boxed{\Rightarrow 0.0119} \longrightarrow ②$$

Compare error predicted formula which is given

$hf''(2)/c$ for some c b/w 2 & 2.1

Since

$f''(x) = 2x^{-3}$ the error

$(0.1)^{-3} \simeq 0.0125$ &

$(0.1)(2.1)^{-3}$

$\simeq 0.0108$

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However the information is usally in not available

Now In second formula $f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(c_1)$

&

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{6} f'''(c_2)$$

where $x-h < c_2 < x < c_1 < x+h$

Substracting the two equation give the three points formula with an explicit error them:

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^2}{12} f'''(1) - \frac{h^2}{12} f'''c_2$$

Hence we will get

$$\boxed{f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^2}{12} f'''(c_1) - \frac{h^2}{12} f'''(c_2)}$$

Required Ans

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Question 2 B Part

Solution:

| | |
|---|---|
| 0 | 1 |
| | 1/2 |
| 2 | 2     $\frac{1}{2}$ |
| | 2 |
| 3 | 3 |

After down x & Y co-ordinates in seperate colum

Calculate the next column

$$\frac{2-1}{2-0} = \frac{1}{2}$$

$$\Rightarrow \frac{2-0\frac{1}{2}}{3-0}$$

$$\Rightarrow \boxed{\frac{1}{2}} \rightarrow 1$$

$$\Rightarrow \frac{4-2}{3-2}$$

$$\Rightarrow \boxed{2} \rightarrow 2$$

after completing the divided difference table the co-effecient of the polonomyl are 1, 1/2, 1/2 as written

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$$P(x) = 1 + \frac{1}{2}(x-0) + \frac{1}{2}(x-0)(x-2)$$

or

In nested form

$$P(x) = 1 + (x-0)\left(\frac{1}{2} + (x-2)\cdot\frac{1}{2}\right)$$

The base point for the nested form $r_1 = 0$ & $r_2 = 2$, then polynomial as

$$P(x) = 1 + \frac{1}{2}x + \frac{1}{2}x(x-2)$$

$$\boxed{P(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 1}$$ Ans is Proved.

x — x — x — x

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Question no 3(a) Part

Solve the least square Problem

$$\begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix}$$

Solution:

According to given condition

$$\begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix}$$

But we know that

The normal equation is

$A^T A x = A^T b$ are

$$\begin{bmatrix} 9 & 6 \\ 6 & 29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 45 \\ 75 \end{bmatrix}$$

The Solution of the normal equation are

$\bar{x}_1 = 3.8$ & $\bar{x}_2 = 1.8$ the residual Vector is

$$r = b - A\bar{x} = \begin{bmatrix} 3 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3.8 \\ 1.8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix} \begin{bmatrix} -3.4 \\ 13 \\ 11.2 \end{bmatrix} = \begin{bmatrix} 0.4 \\ 2 \\ -2.2 \end{bmatrix}.$$

which has Euclidean norm $\|e\|_2 =$

$$\sqrt{(0.4)^2 + 2^2 + (-2.2)^2} = \boxed{3 \text{ Ans}}$$

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Question no 3 B Part

Find the line that best fits the three data points $(-1, y) = (1,2), (-1,1)$ & $(1,3)$ In figure

Solution:

According to given figure

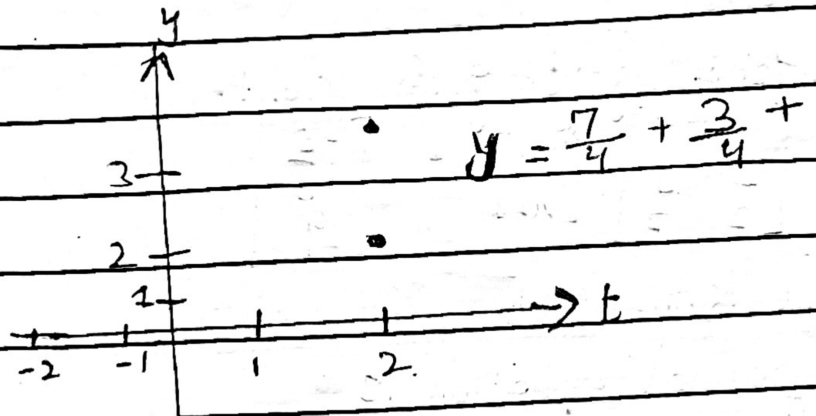


Fig 4.3 Best line example 4.3 one each of the data Point lies above on and below the best line. The model is $y = c_1 + c_2 t$ & the goal is to find the best $C_1$ & $C_2$ subsitution of the point into the model yields

$$c_1 + c_2(1) = 2$$
$$c_1 + c_2(-1) = 1$$
$$c_1 + c_2(1) = 3$$

or In Matrix form is given below

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$ Answer is Proved

$A \quad x \quad b$