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Differential Equation
Final TERM: SUMMER
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⇒ Note: Attempt all Questions

Q1: a

Define 2^{nd} order linear homogenous/non-homogenous differential equations along with two enamples

Ans: 2^{nd} Linear Homogenous differential Equation:
The study of ordinary differential equations of the standard form below, known as the 2^{nd} order linear equation

~~If $g(t) = 0$~~

Examples: $y'' + p(t)y' + q(t)y = g(t)$

If $g(t) = 0$ then the equation above becomes $y'' + p(t)y' + q(t)y = 0$ is

$y'' + 4y' + 5y = 0$

called a homogenous equation.

⇒ 2^{nd} Order Non Homogenous equation:
Non homogenous equation has a corresponding homogenous equation

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$y'' + p(t)y' + q(t)y = 0$ Is Non Homogenous equation

Examples:

$y'' + py' + qy + f(x)$

or

$ay'' + by' + cy = 0$

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"b"

Solve the following 2nd order Linear homogeneous /non homogenous differential equation>

i) $16y'' + 24y' + 9y = 0$

Sol:

$$16y'' + 24y' + 9y = 0$$

Replace

y'' with r^2, y' with r and y with 1

So:

$$16r^2 + 24r + 9 = 0$$

Determine The roots

$$r = \frac{-24 \pm \sqrt{(24)^2 - 4(16)(9)}}{2(16)} = \frac{3}{4}$$

$$y(t) = c_1 e^{rt} + c_2 t e^{rt}$$

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with r the double root of the characteristic equation has only 1 solution.

$$y(t) = c_1 e^{rt} + c_2 t e^{rt}$$

with r the (double) root of the characteristic equation.

Since

$r = -\frac{3}{4}$ the general solution is

$$= \quad y(t) = c_1 e^{-3t/4} + c_2 t e^{-3t/4}$$

Ans

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ii

ii) $y'' - 4y' - 12y = 3e^{n}(5x)$

Sol:

$$y'' - 4y' - 12y = 3e^{(5x)}$$

$$ay'' + by' + cy = g(x)$$

General Solution

$$a(x)y'' + b(x)y' + c(x)y = g(x)$$

$$y = y(a) + y(p)$$

$$y(x) \Rightarrow a(x)y'' + b(x)y + c(x)y = 0$$

$$y'' - 4y' - 12y = 0$$

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$$= \frac{4 \pm \sqrt{16-4(1)(12)}}{2(1)}$$

$$y = C_1 e^{6x} + C_2 e^{-2x}$$

$$y(p) \Rightarrow y'' - 4y' - 12y = 3e^{5x}$$

$$= y = \frac{-3}{7} e^{5x}$$

General solution.

$$y = y(a) + y(p)$$

$$y = C_1 e^{6x} + C_2 e^{-2x} - \frac{3}{7} e^{5x}$$

= = = = = =

Q 2:

Solve the following IVP for the 2nd order linear equations.

(i) $2y'' + 5y' + 3y = 0$ $y(0) = 3$ $y'(0) = -4$

Solution:

subtracting $y = e^{mx}$

$2m^2 + 5m + 3 = 0$ or $m = -1, -3/2$

Hence
the independent solution are

e^{-x}, and $e^{-3x/2}$

$y(x) = Ae^{(-3x)/2} + Be^{-x}$

Initial Condition

$y(0) = A + B = 3$

$y'(0) = -3/2\,A - B = -4 \Rightarrow 3/2\,A + B = 4$

Solving the above equation we get
$\begin{cases} A = 2 \\ B = 1 \end{cases}$

The solution of the initial value problem is

$y(x) = [2e^{-3x/2} + 1e^{-x}]$ Ans

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Q2 (ii)

$2y'' + 5y' - 3y = 0 \quad y(0) = 3 \quad y'(0) = 4$

Solution:

a) $2\,d^2y/dx^2 - 5\,dy/dx - 3y = 0$

b) $2(D^2 + 5D - 3)y = 0;$

substitute $D = m$

$(2m^2 + 5m - 3m)$

$m = 1/2 \; ; \; m = -3$

when:

$x = 0;$

$y = 3;$

$x = 0, \; dy/dx = 4;$

$A + B = 3 \; \ldots \text{ i)}$

$4 = 1/2\,A - 3B \; \ldots \text{ ii)}$

$$A = \frac{26}{7} \; ; \; B = \frac{-5}{7}$$

Hence particular solution is

$$y = \frac{26}{7} e^{1/2 x} - \frac{5}{7} e^{-3x}$$

Ans

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Q2 (iii)

(iii) $y'' - 4y' + 9y = 0$ $y(0) = 0$ $y'(0) = 8$

Solution:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 9y = 0 \quad (D^2 - 4D + 9)y = 0$$

$m^2 - 4m + 9 = 0;$

$m = 2 \pm \sqrt{5}$; → Since the roots are general solutions is:

$$y = e^{2x}(A\cos\sqrt{5}x + B\sin\sqrt{5}x)$$

when

$x = 0;\ y = 0;\quad A = 0;$

Since:

$$y = e^{2x}(A\cos\sqrt{5}x + B\sin\sqrt{5}x)$$

$$+ e^{2x}(-A\sqrt{5}\sin\sqrt{5}x + \sqrt{5}B\cos\sqrt{5}x)$$

$B = -8/\sqrt{5}$ $\qquad -8 = (\sqrt{5})B$

Solution is

$$y = e^{2x}\left(\frac{-8}{\sqrt{5}}\sin\sqrt{5}x\right)$$

Ans.

Q3: A

Q3: Define Laplace transform along with two examples?

Ans: Laplace Transform:
The Laplace transform, named after its inventor Pierre-Simon Laplace is an integral transform that converts a function of a real variable (often time) to a function of a complex variable (complex frequency).

Examples:

$$f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$$

$$h(t) = 3\sinh(2t) + 3\sin(2t)$$

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Q3: 1

$$f(t) = 6e^{\wedge}(-5t) + e^{\wedge}3t + 5(t^{\wedge}3) - 9$$

Sol:

$$f(s) = L\{f(t)\} = 6L\{e^{-5t}\} + L\{e^{3t}\} + 5L\{t^3\} - 9L[1]$$

$$\Rightarrow 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s}$$

$$\frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}$$ Ans

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Q3: 2

$g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

Sol:

$Y(s) = L\{g(t)\}$

$= 4L\{\cos(4t)\} - 9L\{\sin(4t)\} + 2\{\cos(10t)\}$

$= 4\frac{s}{s^2+4^2} - 9\frac{4}{s^2+4^2} + 2\frac{s}{s^2+10^2}$

$\frac{4s}{s^2+4^2} - \frac{36}{s^2+4^2} + \frac{2s}{s^2+10^2}$

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Q3 : 3

$h(t) = e^{n} t + \cos(6t) - e^{n}(3t)\cos(6t)$

Sol:

$g(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$

$L\{e^{3t} + \cos(6t) - e^{3t}\cos(6t)\}$

$= L\{e^{3t}\} + L\{\cos(6t)\} - L\{e^{3t}\cos(6t)\}$

$= \frac{1}{s-3} + \frac{s}{s^2+6^2} + \frac{s-3}{(s-3)^2+36^2}$

$= \frac{1}{s-3} + \frac{s}{s^2+36} + \frac{s-3}{(s-3)^2+36}$

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$)=0$

$)=0$

$\neq$ the

$2Y(s)$

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Q4 (i)

(i) $y'' - 4y' = e^{\wedge}(3t), \quad y(0)=0, \; y'(0)=0$

Sol:

$y'' - 4y' = e^{3t}$

$L\{y'' - 4y'\} = L\{e^{3t}\}$

$L\{y'' - 4y'\} : s^2 L\{y\} - sy(0) - y'(0) - 4$

$sL\{y\} - y(0)$

$L\{e^{3t}\} : \dfrac{1}{s-3}$

$s^2 L\{y\} - sy(0) - y'(0) - 4(sL\{y\} - y(0))$

$= \dfrac{1}{s-3}$

$s^2 L\{y\} - 4sL\{y\} = \dfrac{1}{s-3}$

$y = \dfrac{1}{12} - \dfrac{1}{3} e^{3t} + \dfrac{1}{4} e^{4t}$ Ans

or

$y = L^{-1}\left\{ \dfrac{1}{s(s-3)(s-4)} \right\}$

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Q 4 (ii)

$y'' + 3y' + 2y = e^{-t}$

$y(0) = 0$
$y'(0) = 0$

Solution.

Taking Laplace transform of the differential equation

we obtain.

$[s^2 Y(s) - s(0) - 2] + 3[sY(s) - 0] + 2Y(s)$

$= \frac{1}{s+1}$

$s^2 Y(s) - 2 + 3s(s) + 2(s) = \frac{1}{s+1}$

$Y(s)\{s^2 + 3s + 2\} = \frac{1}{s+1} + 2$

$Y(s)\{s^2 + 3s + 2\} = \frac{1 + 2s + 2}{s+1}$

$Y(s) = \frac{2s+3}{\{(s+1)(s^2+3s+2)\}}$