Department of Electrical Engineering

Assignment

Date: 25/06/2020

Course Details

Course Title: Signals & Systems **Module:** 04

Instructor: Eng Mujtaba Ihsan **Total Marks:** 50

Student Details

Name: M.Salman Shahid **Student ID:** 15006

Q1.	(a) (b)	**Show** with a help of an equation that the differentiation of a function in time domain results in the multiplication by jw in frequency domain. If □[□]= □□[□]−□□[□−□]+□□[□−□] □[□] = □□[□] + □[□ − □] + □□[□ − □] **Produce** Y(z) and y[n]	Marks 06+08 CLO 3
Q2.		□(□) = { −□/□ − □ ≤ □ ≤ □ ; □/□ □ ≤ □ ≤ □ **Retrieve** the Fourier series for the given function.	Marks 10 CLO 3
Q3.		If □(□) = □□□ + □□ / (□□ + □□ − □) **Retrieve** x[n] using inverse Z-transform method.	Marks 10 CLO 3
Q4.		**Express** the transfer function using the given data. A =[−□ −□ ; □ □] B = [□ ; □] C =[□ □] D =[0]	Marks 09 CLO 3
Q5.		**Apply** Fourier transform on the signal, **x (t) = e$^{-a\|t\|}$ u (t)** where u (t) is a unit step function.	Marks 07 CLO 3

Q1.	(a)	**Show** with a help of an equation that the differentiation of a function in time domain results in the multiplication by jw in frequency domain.	Marks 06+08
			CLO 3

Part (a):- Let $x(t)$ be a continuous-time signal with a fourier transform of $X(jw)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(jw)\, e^{jwt}\, dw$$

Differentiate both w.r.t "t"

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(jw)\, \frac{d}{dt}\left\{e^{jwt}\right\} dw$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(jw)\left\{e^{jwt}\cdot jw\right\} dw$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left\{jw\, x(jw)\right\} e^{jwt}\, dw$$

$$F\left\{\frac{d}{dt}x(t)\right\} = jw\, X(jw)$$

So we concluded from this equation that if a function is differentiated in time domain, it is multiplied by jw in frequency domain.

	(b)	If $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$ $h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$ **Produce** Y(z) and y[n]	

Sol:- $x[z] = 2 - 4z^{-2} + 2z^{-3}$

$H[z] = 3 + z^{-1} + 2z^{-2}$

Now $Y(z) = H(z) \times x[z]$

$= (3 + z^{-1} + 2z^{-3}) \times (2 - 4z^{-2} + 2z^{-3})$

$Y(z) = 6 - 12z^{-2} + 6z^{-3} + 2z^{-1} - 4z^{-3} + 2z^{-4} + 4z^{-2} - 8z^{-4} + 4z^{-5}$

$Y(z) = 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$

To find $y[n]$ use the delay property

$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] + 2\delta[n-3] - 6\delta[n-4] + 4\delta[n-5]$

Q2.		$f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$ **Retrieve** the Fourier series for the given function.	Marks 10 CLO 3

Let $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx \, dx$

$= \frac{1}{\pi}\int_{-\pi}^{0} f(x) \cos nx \, dx + \int_{0}^{\pi} f(x) \cos nx \, dx$

$= \frac{1}{\pi}\int_{-\pi}^{0} -\frac{\pi}{2} \cos nx \, dx + \int_{0}^{\pi} \pi/2 \cos nx \, dx$

$= \frac{1}{\pi}\left(\pi/2 \frac{\sin nx}{n} \Big|_{-\pi}^{0} + \pi/2 \frac{\sin nx}{n} \Big|_{0}^{\pi} \right.$

$= \frac{1}{n\pi}\left[\frac{-\pi}{2} \sin n(0) - \sin n(-\pi) \right]$

$+ \pi/2 \left[\sin n(\pi) - \sin n(0) \right]$

$= \frac{1}{n\pi}(0)$

$a_n = 0$

Now

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin nx \, dx$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_{0}^{\pi} f(x)\sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} \frac{-\pi}{2}\sin nx\,dx + \int_{0}^{\pi} \pi/2\,\sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[-\pi/2 \int_{-\pi}^{0} \sin nx\,dx + \pi/2 \int_{0}^{\pi} \sin nx\,dx \right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{2}\, \frac{-\cos nx}{n}\Big|_{-\pi}^{0} + \frac{\pi}{2}\, \frac{-\cos nx}{n}\Big|_{0}^{\pi} \right]$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}\left[-\cos n(0) + \cos n(-\pi)\right] + \pi/2 \left[-\cos n(\pi) + \cos n(0)\right] \right]$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}\left[-1 + \cos n(-\pi)\right] + \pi/2 \left(-\cos n\pi + \cos n(0)\right) \right]$$

$$= \frac{\pi}{2} = \frac{1}{n\pi}\left\{ \frac{-\pi}{2} - 1\left[-1 + \cos n(-\pi)\right] + 1\left[-\cos n\pi + 1\right] \right.$$

$$= \frac{1}{2\pi}\left[1 - \cos n\pi - \cos n\pi + 1\right]$$

$$= \frac{1}{2n}\left[2 - 2\cos n\pi\right]$$

Now
$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{4}{2n} & \text{if } n \text{ is odd} \end{cases}$$

$$b_n = \frac{4}{2n}$$

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \cdots$$
$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \cdots$$

$$f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$$

$$a_0 = \frac{1}{2\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_0^{\pi} f(x)\,dx\right]$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} \frac{-\pi}{2}\int_{-\pi}^{0} dx + \int_0^{\pi} \frac{x}{2}\,dx$$

$$= \frac{1}{2\pi}\left[\frac{-\pi}{2}\int_{-\pi}^{0} 2\,dx + \frac{\pi}{2}\int_0^{\pi} 1\,dx\right]$$

$$= \frac{1}{2\pi}\left[\frac{-\pi}{2}\,x\Big|_{-\pi}^{0} + \frac{\pi}{2}\,x\Big|_0^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\pi/2\left[0-(-\pi)\right) + \frac{\pi}{2}\left[\pi-0\right]\right]$$

$$= \frac{1}{2\pi}\left[\frac{-\pi}{2}(\pi) + \pi/2\,(\pi)\right]$$

$$= \frac{1}{2\pi}\left\{\frac{-\pi^2}{2} + \frac{\pi^2}{2}\right\}$$

$$= \frac{1}{2\pi}\left[\frac{0}{2}\right] = \quad a_0 => 0$$

Q3.		If $X(z) = \frac{2z^2 + 2z}{(z^2 + 2z - 3)}$ **Retrieve** x[n] using inverse Z-transform method.	Marks 10 CLO 3

$$x(z) = \frac{2z^2+2z}{z^2+2z-3}$$

$$= \quad x(z) = \frac{2z(z+1)}{z^2+2z-3}$$

$$= \quad \frac{x(z)}{2z} = \frac{z+1}{(z-1)(z+3)}$$

$$\frac{2z+2}{(z-1)(z+3)} = \frac{A}{(z-1)} + \frac{B}{(z-3)} \quad \text{—(i)}$$

$$= 2z+2 = A(z+3) + B(z-1) \quad \text{—(ii)}$$

Puting $z=-3$ in eq (ii)

$$2(-3)+2 = A(-3+3) + B(-3-1)$$

$$-6+2 = A(0) + B(-4)$$

$$\frac{-4}{-4} = \frac{B(-4)}{-4}$$

$$B = 1$$

Now Put $z=1$ in eq ii

$$2(1) + 2 = A(1+3) + B(1-1)$$

$$2+2 = A(4) + B(0)$$

$$4 = A4 + 0$$

$$A = 1$$

Now put value of A & B in eq (i)

$$\frac{2z+2}{(z-1)(z+3)} = \frac{1}{(z-1)} + \frac{1}{(z-3)}$$

$$x(z) = \frac{z}{z-1} + \frac{z}{z+3}$$

Inverse z - transform these are not in a table so

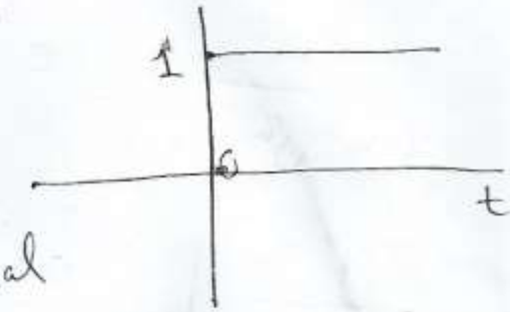
$$x[n] = \frac{z}{z-1} + \frac{z}{z+3}$$

$$f(k) = u[k] + 1 \cdot 3^{k}$$

Q5.		**Apply** Fourier transform on the signal, $\mathbf{x(t) = e^{-a|t|} u(t)}$ where u (t) is a unit step function.	Marks 07 CLO 3

$x(t) = e^{-a|t|} \, u(t)$

Solution :- $x(j\omega) = ?$

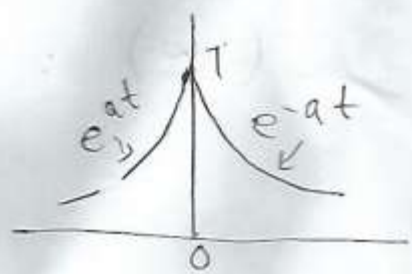


Continuous - Time fourier Transform of a signal $x(t)$ is given by

$$x(j\omega) = \int_{-\infty}^{\infty} x(t) \, e^{-j\omega t} \, dt$$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|} \, e^{-j\omega t} \, dt$$

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t > 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$



$$x(j\omega) = \int_{-\infty}^{0} e^{at} \, e^{-j\omega t} \, dt + \int_{0}^{\infty} e^{-at} \, e^{-j\omega t} \, dt$$

$$\omega = \int_{-\infty}^{0} e^{(a-j\omega)t} \, dt + \int_{0}^{\infty} e^{-(a+j\omega)t} \, dt$$

$$= \frac{e^{(a-jw)t}}{a-jw}\Big|_{-\infty}^{0} + \frac{e^{-(a+jw)t}}{-(a+jw)}\Big|_{0}^{\infty}$$

$$= \frac{1}{(a-jw)}\left[e^{0}-e^{-\infty}\right] - \frac{1}{(a+jw)}\left[e^{-\infty}-e^{0}\right]$$

$$= \frac{1}{(a-jw)}\left[1-0\right] - \frac{1}{(a+jw)}\left[0-1\right]$$

$$= \frac{1}{(a-jw)} + \frac{1}{a+jw}$$

$$= \frac{a+jw+a-jw}{a^{2}-(jw)^{2}}$$

$$(jw) = \frac{2a}{a^{2}+w^{2}}$$

Q4.		Express the transfer function using the given data. $A=\begin{bmatrix}-2 & -1\\ 1 & 0\end{bmatrix}$ $B=\begin{bmatrix}1\\0\end{bmatrix}$ $C=[1 \quad 2]$ $D=[0]$	Marks 09 / CLO 3

Q4:-

$A=\begin{bmatrix}-2 & -1\\ 1 & 0\end{bmatrix}$ $B=\begin{bmatrix}1\\0\end{bmatrix}$ $C=[1 \quad 2]$ $D=[0]$

Sol:-

$$G(s) = C(sI-A)^{-1}B+D$$

$$= [1\ 2]\left[s\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}-2 & -1\\ 1 & 0\end{bmatrix}\right]^{-1}\begin{bmatrix}1\\0\end{bmatrix}+[0]$$

$$= [1\ 2]\left[\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix}-2 & -1\\ 1 & 0\end{bmatrix}\right]^{-1}\begin{bmatrix}1\\0\end{bmatrix}+[0]$$

$$= [1\ 2]\begin{bmatrix}s+2 & 1\\ -1 & s\end{bmatrix}^{-1}\begin{bmatrix}1\\0\end{bmatrix}+[0]$$

$$= [1\ 2]\frac{1}{s(s+2)+1}\times\begin{bmatrix}s & -1\\ 1 & s+2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$$

$$= \frac{1}{s^2+2s+1}[1\ 2]\begin{bmatrix}s\\1\end{bmatrix}$$

$$= \frac{1}{s^2+2s+1}[s\ 2]$$