(1)

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13741 Numerical Analysis

Q No: 1 (a)

Sol: The lagrange form is:

$\cancel{P(x)}$ $l_1(x) = \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)}$

$= \frac{-1}{6}[x-1)(x-2)(x-3)$

$l_2(x) = \frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}$

$= \frac{1}{2}(x-0)(x-2)(x-3)$

$l_4(x) = \frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)} = \frac{1}{6}(x-0)(x-1)(x-2)$

$l_3(x) = \frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)} = \frac{-1}{2}(x-0)(x-1)(x-3)$

$P(x) = 2\left[-\frac{1}{6}(x-1)(x-2)(x-3)\right] + 1\left[\frac{1}{2}(x-0)(x-2)(x-3)\right.$

$-1\left[\frac{1}{6}(x-0)(x-1)(x-2)\right]$

2

$$P(x) = -\frac{1}{3}\left(x^3 - 6x^2 + 11x - 6\right) + \frac{1}{2}\left(x^3 - 5x^2 + 6x\right) - \frac{1}{6}\left(x^3 - 3x^2 + 2x\right)$$

$$P(x) = -x + 2$$

X ———————————— X

Q1 (b)

Find an Interpolating Polynomial for the ....

Sol: Subtituting Into lagrange,s formula

$$P_2(x) = 1\left(\frac{(x-2)(x-3)}{(0-2)(0-3)}\right] + 2\left(\frac{(x-0)(x-3)}{(2-0)(2-3)}\right.$$

$$+ 4\left(\frac{x-0)\ (x-2)}{(3-0)\ (3-2)}\right.$$

$$= \frac{1}{6}(x^2-5x+6) + 2\left(\frac{-1}{2}\right)(x^2-3x)$$

$$+ 4\left(\frac{1}{3}\right)(x^2-2x)$$

$$= \frac{1}{2}x^2 - \frac{1}{2}x + 1$$

Check that $P_2(0) = 1$, $P_2(2) = 2$ and $P_2(3) = 4$

In General.

$$L_k(x) = \frac{(x-x_1)\ \ldots\ x-x_{k-1})\ (x-x_{k+1}\ldots x-x_n)}{(x_k-x_1)\ (x_k-x_{k-1})\ \ldots\ (x_k-x_n)}$$

Q 2(a)

Use the two point farward Difference

Sol: The two Point farward-differnce formula evaluates to:

$$f'(x) \approx \frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{2.1}-\frac{1}{2}}{0.1}$$

$$\approx -0.2381$$

The difference b/w this approximation and the correct derivative $f'(x)$.

$f'(x) = -x^{-2}$ at $x = 2$ is the error-

$-0.2381 - (-0.2500) = 0.0119.$

Compare this to error Predicted by. the formula.

$hf''(c)/2$ for some c b/w 2 and 2.1 since $f''(x) = 2x^{-3}$

The error Must be b/w.

$(0.1)2^{-3} \approx 0.0125$ and $(0.1)(2.1)^{-3} \approx 0.0108$

Which is consistant with our results.

A second Order formula is.

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(c_1)$$

and

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{6} f'''(c_2)$$

where $x - h < c_2 < x < c_1 < x + h$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^2}{12} f'''(c_1) - \frac{h^2}{12} f'''(c_2)$$

x ———————— x

Q 2 (b)

Use divided difference to find the Interpolating - - - - - (0,1) ,(2, 2), (3, 4)

Apply the definitions of divided differences leads to the following table.

$$\begin{array}{c|cccc} 0 & 1 & & & \\ & & \frac{1}{2} & & \\ 2 & 2 & & \frac{1}{2} & \\ 3 & 3 & 2 & & \end{array}$$

After writing down the x and y Coordinates in separate Coloumns, Calculate the next colomns. as divided differences.

$$\frac{2-1}{2-0} = \frac{1}{2}$$

$$\frac{2-\frac{1}{2}}{3-0} = \frac{1}{2}$$

$$\frac{4-2}{3-2} = 2$$

After completing the divided difference triangle.

$$P(x) = 1 + \frac{1}{2}(x-0) + \frac{1}{2}\cancel{x}(x-0)(x-2)$$

or In nested form.

$$P(x) = 1 + (x-0)\left(\frac{1}{2} + x - 2 \cdot \frac{1}{2}\right)$$

The base points for the nested form are $r_1 = 0$ and $r_2 = 2$

We can write the Interpolating Polynomial as:

$$P(x) = 1 + \frac{1}{2}x + \frac{1}{2}x(x-2)$$

$$P(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 1$$

QNo: 3 (a)

Solve the least squares problem.

$$\begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix}$$

The normal equations $A^T A x = A^T b$ are:

$$\begin{bmatrix} 9 & 6 \\ 6 & 29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 45 \\ 75 \end{bmatrix}$$

The solution of the normal equations are $\bar{x}_1 = 3.8$ and $\bar{x}_2 = 1.8$ The residual vector is.

$$r = b - A\bar{x} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3.8 \\ 1.8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} -3.4 \\ 13 \\ 11.2 \end{bmatrix} = \begin{bmatrix} 0.4 \\ 2 \\ -2.2 \end{bmatrix}$$

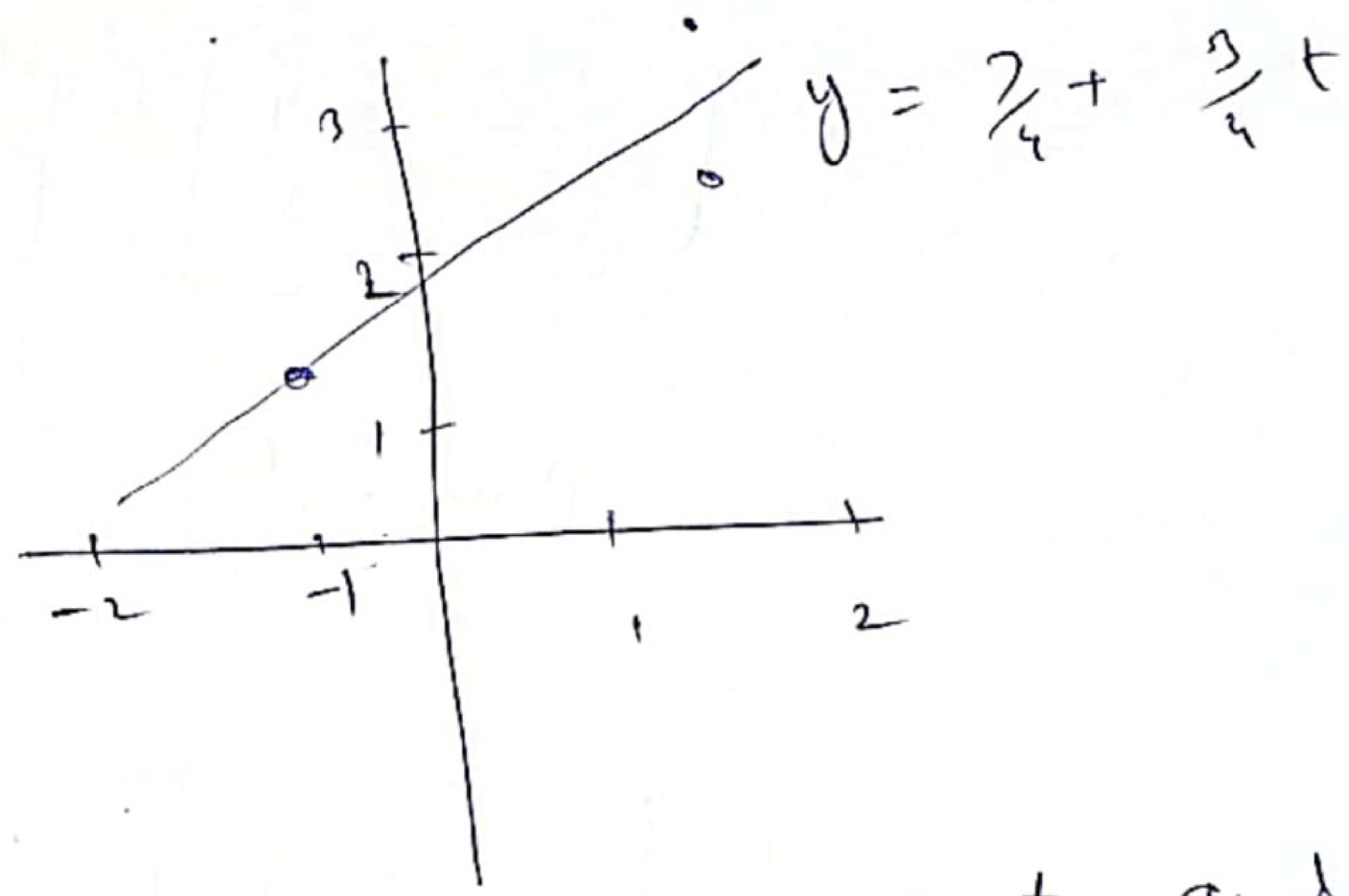
which has Euclidean norm $\|e\|_2$

$$= \sqrt{(0.4)^2 + (2)^2 + (-2.2)^2} = 3$$

x————————x

9

Find the line that best fits Three data points $(t, y) = (1, 2)$ $(-1, 1)$ and $(1, 3)$



Sol: The Model is $y = c_1 + c_2 t$. and the Goal is to find the best c_1 and c_2. Subtituting of the data Points into the Model yields.

$$c_1 + c_2 (1) = 2$$

$$c_1 + c_2 (-1) = 1$$

$$c_1 + c_2 (1) = 3$$

Or In Matrix form:

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

X ——— X

The end.