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7894

Sec A

Differential Equation.

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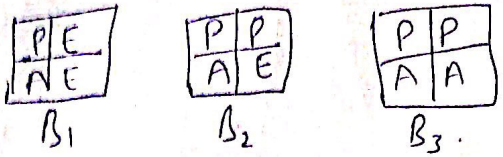
A yarn merchant sells brands A, B, C of yarn each of which is a blend of Pakistani egyptian and American cotton in the ratios of 1:2:1 ; 2:1:1 ; 2:0:2 respectively. If one kilogram of A, B, C cost 40, 50, 60 rupees respectively. find the cost of kilogram of cotton of each variety.

Solution:

1 : 2 : 1 ; 2 : 1 : 1 ; 2 : 0 : 2.



B_1 B_2 B_3.

Let a, b, and c be the cost per kg of Pakistani, egyptian and american cotton. According to the given conditions.

$$\left.\begin{aligned} \frac{1}{4}a + \frac{2}{4}b + \frac{1}{4}c &= 40 \\ \frac{2}{4}a + \frac{1}{4}b + \frac{1}{4}c &= 50 \\ \frac{2}{4}a + \frac{0}{4}b + \frac{2}{4}c &= 60 \end{aligned}\right] \Rightarrow A.$$

$$\left.\begin{aligned} 1a + 2a + 1c &= 160 \\ 2a + 1b + 1c &= 200 \\ 1a + 1c &= 120 \end{aligned}\right] \Rightarrow B.$$

Matrix form.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad B = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}.$$

$$AX = B$$

$$A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} \quad \text{Expanding By } R_1$$

$$|A| = 1\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(1\times1-0) - 2(2\times1-1\times1) + 1(2\times0-1).$$

$$= 1 - 2 + 1(-1) = -2 \Rightarrow |A| = -2 \neq 0$$

$$|A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix}$$ Expanding By R_1.

$$|A_1| = 160\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1\begin{vmatrix} 200 & 1 \\ 120 & 0 \end{vmatrix}$$

$$= 160(1\times1 - 0\times1) - 2(200\times1 - 120\times1) + 1(200\times0 - 120\times1)$$

$$|A_1| = 160 - 160 - 120$$

$$|A_1| = -120.$$

$$|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix}$$ Expanding by R_1

$$|A_2| = 1\begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} - 160\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix}$$

$$= 1(200\times1 - 120\times1) - 160(2\times1 - 1\times1) + 1(2\times120 - 1\times200)$$

$$= 80 - 160 + 40.$$

$$|A_2| = -40$$

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$$|A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix}$$ Expanding by R_1

$$|A_3| = 1\begin{vmatrix} 1 & 200 \\ 0 & 120 \end{vmatrix} - 2\begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix} + 160\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(120 - 0 \times 200) - 2(2 \times 120 - 1 \times 200) + 160(2 \times 0 - 1 \times 1).$$

$$= 1(120) - 2(40) + 160(-1).$$

$$= 120 - 80 - 160$$

$$|A_3| = -120.$$

Now By Cramer's Rule.

$$a = \frac{|A_1|}{|A|} = \frac{60}{-2} \Rightarrow a = 60.$$

$$b = \frac{|A_2|}{|A|} = \frac{-40}{-2} \Rightarrow b = 20.$$

$$c = \frac{|A_3|}{|A|} = \frac{60}{-2} \Rightarrow c = 60.$$

$$(a, b, c) = (60, 20, 60)$$

Pakistan = 60
Egyptian = 20
American = 60.