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Question 1:

Use any of methodes for solving the ordinary Differential equation. As given below:

Solve and graph the solution show the details of your work.

$x^2y'' - 4xy' + 6y = 0 \quad y(1) = 0.4,$
$y'(1) = 0$

Solution:

$$x^2y'' - 4xy' + 6y = 0 \quad \text{---(1)}$$

But $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

put $y'' = m(m-1)x^{m-2}$ in eq(1)

$x^2 m(m-1)x^{m-2} - 4x m x^{m-1} + 6x^m = 0$

$\cancel{x^2} m(m-1)x^m \cancel{x^{-2}} - 4\cancel{x} m x^m \cdot \cancel{x^{-1}}$
$+ 6x^m = 0$

$x^m[m(m-1) - 4m + 6] = 0$

$\Rightarrow m(m-1) - 4m + 6 = 0$

$m^2 - 5m + 6 = 0$

$m^2 - 5m + 6$

$$m = \frac{5 \pm \sqrt{(-5)^2 + 4(6)}}{2a}$$

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$\sim = \frac{5+1}{2}$, $\sim = \frac{5-1}{2}$

$\sim = \frac{6}{2}$, $\sim = \frac{4}{2}$

$\sim = 3$, $\sim = 2$

So

$y' = x^3$ $\quad$ $y^2 = x^2$

$y = c_1 y_1 + c_2 y_2$

$y = c_1 x^3 + c_2 x^2$

$y' = 3c_1 x^2 + 2c_2 x$

putting values of y and y'

$\Rightarrow$ $0.4 = c_1 1^3 + c_2 1^2$

$0 = 3c_1 1^3 + 2c_2 1^2$

$\Rightarrow$ $0.4 = c_1 + c_2$

$0 = 3c_1 + 2c_2$

$\Rightarrow$ $0.4 - c_2 = c_1$

$0 = 3(0.4 - c_2) + 2c_2$

$\Rightarrow$ $0.4 - c_2 = c_1$

$1.2 = c_2$

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$\Rightarrow \quad 0.4 - 1.2 = c_1$

$c_2 = 1.2$

$c_1 = -0.8$

$c_2 = 1.2$

$\boxed{y = (-0.8 \times 3) + 1.2 \times 2}$ Ans

Question 13

$x^2y'' + 3xy' + 0.75y = 0$

$y(1) = 1, \quad y'(1) = 1.5$

Solution :-

$x^2y'' + 3xy' + 0.75y = 0$ —(i)

$y = x^m$

$y' = mx^{m-1}$

$y'' = (m-1)x^{m-2}$

$x^2m(m-1)x^{m-2} + 3xmx^{m-1} + 0.75x^m = 0$

$\Rightarrow \cancel{x^2}m(m-1)x^m\cancel{x^{-2}} + 3\cancel{x}mx^m \cdot \cancel{x^{-1}} + 0.75x^m = 0$

$\Rightarrow x^m[m(m-1) + 3m + 0.75] = 0$

$m^2 + 2m + 0.75 = 0$

$m_{1/2} = \dfrac{-2 \pm \sqrt{2^2 - 4(0.75)}}{2}$

$m_{1/2} = \dfrac{-2 \pm 1}{2}$

$m_1 = \dfrac{-2+1}{2}, \quad m_2 = \dfrac{-2-1}{2}$

$m_1 = \dfrac{-1}{2}, \quad m_2 = \dfrac{-3}{2}$

$y = c_1 y_1 + c_2 y_2$

$y = c_1 x^{0.5} + c_2 x^{-1.5}$

$y' = 0.5 c_1 x^{-1.5} - 1.5 c_2 x^{-2.5}$

$y(1) = 1 = c_1 1^{-0.5} + c_2 \cdot 1^{-1.5}$

$1.5 = y'(1) = 0.5 c_1 1^{1.5} - 1.5$
$c_2 1^{-2.5}$

$\Rightarrow \quad 1 = c_1 + c_2$
$\quad 1.5 = 0.5 c_1 - 1.5 c_2$

$\Rightarrow \quad 1 = c_1 + c_2$
$\quad 3 = c_1 + 3 c_2$

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$\Rightarrow$ $1 - c_2 = c_1$

$3 = 1 - c_2 + 3c_2$

$\Rightarrow$ $1 - c_2 = c_1$

$2 = 2c_2$

$\Rightarrow$ $1 - c_2 = c$

$1 = c_2$

$0 = c_1$

$1 = c_2$

$c_1 = 0$

$c_2 = 1$

$$\boxed{y^2 = x^{-1.5}}$$

Question 14

$x^2y'' + xy' + 9y = 0$

$y(1) = 0, \quad y'(1) = 2.5$

Solution :-

$x^2y'' + xy' + 9y = 0$

$y = x^m$

$y' = m x^{m-1}$

$y'' = (m-1)m x^{m-2}$

$\cancel{x^2} m(m-1)x^m \cancel{x^{-2}} + m x^m + 9x^m = 0$

$-(m-1)+m+9=0$

$m^2-m+m-9=0$

$m^2+9=0$

$m^2-(3i)^2=0$

$(m-3i)(m+3i)=0$

$m_1=3i \wedge m_2=-3i$

putting values

$x^{m_1}=x^{3i}=(e^{\ln x})^{3i}=e^{3i\ln x}$

$x^{m_2}=x^{-3i}=(e^{\ln x})^{-3i}=e^{-3i\ln x}$

$e^{a}=e^{a+b}=e^{a}(\cos b+i\sin b)$

$\Rightarrow e^{3i\ln x}=e^{0}[\cos(3\ln x)+i\sin(3\ln x)]$

$\Rightarrow \cos(3\ln x)+i\sin(3\ln x)$

$\Rightarrow e^{-3i\ln x}=e^{0}[\cos(3\ln x)-i\sin(3\ln x)]$

$=\cos(3\ln x)-i\sin(3\ln x)$

$x^{m_1}=\cos(3\ln x)+i\sin(3\ln x)$

$x^{m_2}=\cos(3\ln x)-i\sin(3\ln x)$

By Adding

$x^{m_1}-x^{m_2}=\cancel{\cos(3\ln x)}+i\sin(3\ln x)-$

$\cancel{\cos(3\ln x)}+i\sin 3\ln x$

$\dfrac{x^{m_1}-x^{m_2}}{2i}=\dfrac{\cancel{2i}\sin(3\ln x)}{\cancel{2i}}$

$=\sin 3\ln x$

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Solution will be

$y_1 = \cos(3\ln x)$, $y_2 = \sin 3\ln x$

$y = c_1\cos(3\ln x + c_2\sin 3\ln x$

$y_1 = c_1\cos 3\ln x + c_2\sin(\cancel{3\ln 1}) = 0$

$\Rightarrow c_1 = 0$

$y'(1) = -3c_1\sin\frac{(\cancel{3\ln 1})}{1} + 3c_2 \cos\frac{3(\cancel{\ln 1})}{1}$

$= 2.5$

$c_1 = 0$

$c_2 = \frac{5}{6}$

$\boxed{y = \frac{5}{6}\sin 3\ln x}$ A

Question 15

$x^2y'' + 3xy' + y = 0$

$y(1) = 36$, $y'(1) = 04$

Solution :-

$x^2y'' - 3xy' + y = 0$

$y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1)x^{m-2}$

$\Rightarrow x^2 m(m-1)x^{m}x^{-2} + 3m x^{m} + x^{m} = 0$

$m(m-1) + 3m + 1 = 0$

$m^2 + 2m + 1 = 0$

$m^2 + 2m + 1 = 0$

$(m+1)^2 = 0$

$m = -1$

$y_1 = x^m = x^{-1} = \frac{1}{x}$

$y'' + \frac{3}{x} \cdot y' + \frac{1}{x^2} \cdot y = 0$

$P(x) = 3\frac{1}{x} \Rightarrow \int P\,dx = 3\ln x$

$y_2 = vy$

$v = \int u\,dx \quad , \quad u = \frac{1}{y^2} e^{-\int p\,dx}$

$e^{-3\ln x} = (e^{\ln x})^{-3} = x^{-3}$

$u = x^{-3} \cdot \frac{1}{x^2} = x^{-3+2} = x^{-1} = \frac{1}{x}$

By integration

$v = \int \frac{dx}{x} = \ln x$

$y_2 = v y_1 = y_1 \ln x = \frac{1}{x} \cdot \ln x$

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Since CE

$y = C_1 y_1 + C_2 y_2$

$\Rightarrow C_1 \frac{1}{x} + C_2 \frac{1}{x} \ln x$

$\Rightarrow \frac{1}{x}(C_1 + C_2 \ln x)$

$\Rightarrow y' = (x^{-1})'(C_1 + C_2 \ln x) + x^{-1}(C_1 + C_2 \ln x)'$

$\Rightarrow -x^{-2}(C_1 + C_2 \ln x) + \frac{1}{x} C_2 \cdot \frac{1}{x}$

$\Rightarrow \frac{1}{x^2}(C_1 + C_2 \ln x + \frac{1}{x} C_2 \cdot \frac{1}{x}$

$\Rightarrow \frac{1}{x}(-C_1 - C_2 \ln x + C_2)$

Values of C

$3.6 = y(1) = \frac{1}{1}(C_1 + C_2 \ln x)$

$0.4 = y'(1) = \frac{1}{1^2}(-C_1 - C_2 \ln 1 + C_2)$

$3.6 = C_1$

$0.4 = C_1 + C_2$

$3.6 = C_1$

$0.4 = -3.6 + C_2$

$3.6 = C_1$

$4.0 = C_2$

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$$\boxed{y = (3c_1 + \ldots \ln x)\cdot \frac{1}{x}}$$ Ans

Question 16

$(x^2D^2 - 3xD + 4I)\,y = 0$

$y(1) = -\pi,\quad y'(1) = 2\pi$

Solution:-

$(x^2D^2 - 3xD + 4I)\,y = 0$

$\Rightarrow x^2 D(Dy) - 3xDy + 4y$

$\Rightarrow x^2y'' - 3xy' + 4y = 0$

$y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1)x^{m-2}$

$x^2 m(m-1)x^{m-2} - 3x\,m x^{m-1} + 4x^m = 0$

$x^2 m(m-1)x^m x^{-2} - 3x m x^m x^{-1} + 4x^m =$

$\Rightarrow m(m-1) - 3m + 4 = 0$

$\Rightarrow m^2 - 4m + 4 = 0$

$\Rightarrow (m-2)^2 = 0$

$m = 2$

$y_1 = x^m = x^2$

$y'' = \frac{3}{x}\cdot y' + \frac{4}{x^2}\cdot y = 0$

$P(x) = -3\cdot\frac{1}{x} \Rightarrow \int P\,dx = -3\ln(x)$

Put $y_2 = v y_1$

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where

$v = \int v\,dx$, $v = \frac{1}{y_1^2} e^{-\int p\,dx}$

$e^{-\int p\,dx} = e^{3\ln|x|} = (e^{\ln|x|}) = x^3$

$v = x^3 \frac{1}{(x^2)^2} = x^{3-4} = x^{-1} = \frac{1}{x}$

$v = \int \frac{dx}{x} = \ln|x|$

$\boxed{y_2 = v y_1 = y_1 \ln x = x^2}$ A

$y = c_1 y_1 + c_2 y_2$

$= c_1 x^2 + x^2 \ln x$

$= x^2 (c_1 + c_2 \ln x)$

Product Rule

$y' = (x^2)'(c_1 + c_2 \ln x) + x^2 (c_1 + c_2 \ln x)'$

$= 2x(c_1 + c_2 \ln x) + c_2 x^2 \cdot \frac{1}{x}$

$= 2c_1 + 2c_2 x \ln x + c_2 x$

$= 2c_1 x + c_2 x (2\ln x + 1)$

Finding C

$-\pi = y(1) = (1)^2 (c_1 + c_2 \ln 1)$

$2\pi = y'(1) = 2c_1 + c_2(2\ln 1 + 1)$

$-\pi = c_1$

$2\pi = 2c_1 + c_2$.

$-\pi = c_1$

$4\pi = c_2$

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$\boxed{y = x^2(-\pi + 4\pi - (m\pi))}$

Question 17

$(x^2D^2 + xD + I)\, y = 0$

$y(1) = 1, \quad y'(1) = 1$

Solution:

$x^2D^2 + xD + I)\, y = 0$

$x^2 D(Dy) + xDy + y$

$= x^2 y'' + xy' + y$

Now

$x^2y'' + xy' + y = 0$

$y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1) x^{m-2}$

Putting values

$x^2 m(m-1) x^{m-2} + x m x^{m-1} +$

$x^m = 0$

$x^2 m(m-1) x^m = x^2 + x m x$

$x^{-1} + x^m = 0$

$m(m-1) + m + 1 = 0$

$m^2 - m + m + 1 = 0$

$m^2 + 1 = 0$

$m^2 + i^2 = 0$

$(m - i)(m + i) = 0$

$m_1 = i, \quad m_2 = i$

$x = e^{\ln x}$

$x^{m_1} = x^{i} = (e^{\ln x})^{i} = e^{i \ln x}$

$e^{z} = e^{a+ib} = e^{a}(\cos b + i\sin b)\, z \in C$

$e^{i \ln x} = e^{0}\{\cos(\ln x) + i\sin(\ln x)\}$

$\Rightarrow \cos(\ln x)\, i\sin(\ln x)$

$x^{m_1} = \cos(\ln x) + i\sin(\ln x)$

$x^{m_2} = \cos(\ln x) - i\sin(\ln x)$

$\dfrac{x^{m_1} + x^{m_2}}{2} = \dfrac{\cancel{2}\cos(\ln x)}{\cancel{2}} =$

$\cos(\ln x)$

$x^{m_1} + x^{m_2} = \cos(\ln x) + i\sin(\ln x)$

$- \cos(\ln x) + \sin(\ln x)$

$\Rightarrow 2i\sin(\ln x)$

$\dfrac{x^{m_1} + x^{m_2}}{2} = \sin(\ln x)$

$y = c_1 y_1 + c_2 y_2$

$\Rightarrow (c_1\cos(\ln x) + 2\sin(\ln x)$

Using chain Rule

$y' = -c_1\sin(\ln x)\cdot(\ln x)' + c_2\cos(\ln x)\cdot(\ln x)'$

$= -\dfrac{c_1}{x}\sin(\ln x) + \dfrac{c_2}{x}\cos(\ln x)$

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Value of "C"

$1 = y(1) = c_1 \cos(\ln x) + c_1 \sin(\ln x)$

$1 = y(1) = \sin(\ln x + 3c_2 \cos(\ln x))$

$1 = c_2$

$1 = c_2$

$$\boxed{y = \sin(\ln x) + \cos(\ln x)}$$ Ans

Question 18

$(9x^2D^2 + 3xD + I)y = 0$

$y(1) = 1$

$y'(1) = 0$

Solution:-

$(9x^2D^2 + 3xD + I)y = 0$

$\Rightarrow 9x^2y'' + 3xy' + y$

$y = x^m$

$y' = mx^{m-1}$

$y'' = (m-1)m\,x^{m-2}$

$9x^2m(m+1)x^{m-2} + 3xmx^{m-1}$

$+ x^m = 0$

$\cancel{9x^2}m(m-1)x^m \cancel{x^{-2}} + 3xmx$

$m\,x\,\cancel{x}^{-1} + x^m = 0$

$9m(m+1) + 3m + 1 = 0$

$9m^2 + 9m + 3m + 1 = 0$

$m^2 - 4m + 4 = 0$

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$(3m-1)^2 = 0$

$9m^2 - 6m + 1 = 0$

$m_1, m_2 = \frac{6 \pm \sqrt{6^2 - 4 \cdot 9}}{18}$

$= \frac{6}{18}$

$m_1 = m_2 = \frac{1}{3}$

$m = \frac{1}{3}$

$y'' + \frac{1}{3} \cdot \frac{1}{x} \cdot y' + \frac{1}{9x^2} \cdot y = 0$

$P(x) = \frac{1}{3} \cdot \frac{1}{x} \Rightarrow \int P dx = \frac{1}{3} \ln x$

put $y = c_1 y_1$

$v = \int V dx \quad \pi \quad V = \frac{1}{y_1^2} e^{-\int P dx}$

$e^{-\int P dx} = e^{-1/3 \ln |x|} = (e^{\ln |x|})^{-1/3} = x^{-1/3}$

$V = x^{-1/3} \cdot \frac{1}{(x^{-1/3})^2} = x^{-1/3 - 2/3} = x^{-1} = \frac{1}{x}$

$y = c_1 y_1 + c_2 y_2$

$y = c_1 x^{1/3} + x^{1/3} \ln x$

$y = x^{1/3} (c_1 + c_2 \ln x)$

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Now

$1 = y(1) = 1^{1/3}(c_1 + c_2 \ln 1)$

$0 = y'(1) = \frac{1}{3} 1^{-2/3}(c_1 + c_2 \ln 1) + 1^{1/3}$

$1 = c_1$

$0 = \frac{c_1}{3} + c_2$

$1 = c_1$

$c_2 = -\frac{1}{3}$

$y = x^{1/3}(1 - \frac{1}{3}\ln x)$ Ans

Question 19

$(x^2D^2 - xD - 15I)y = 0$

$y(1) = 0.1$

$y'(1) = -4.5$

Solution :-

$y = x^m$

$y' = mx^{m-1}$

$y'' = (m-1)mx^{m-2}$

$m^2 - 2m - 15 = 0$

$m_{1/2} = \frac{2 \pm \sqrt{(-2)^2 + 4 \cdot 1 \cdot 15}}{2}$

$m_{1/2} = \frac{2 \pm 2}{2}$

$m_1 = 5, \quad m_2 = -3$

$0.1 = y(1) = c_1\, 1^5 + c_2\, 1^{-3}$

$-4.5 = y'(1) = 5c_1 - 1^4 - 3c_2 - 1^{-4}$

$0.1 = c_1 + c_2$

$-4.5 = 5c_1 - 5c_2$

$0.1 - c_2 = c_1$

$-4.5 = 5(0.1 - c_2) - 3c_2$

$-0.525 = c_1$

$0.625 \quad c_2$

$\Rightarrow y = -0.525x^5 + 0.625x^{-3}$ A

Question 2

$x' = \sqrt{x}$

Solution

$x' = \sqrt{x}$

$\Rightarrow \dfrac{dx}{dt} = \sqrt{x}$

$\Rightarrow \dfrac{dx}{\sqrt{x}} = 1\,dt$

$\Rightarrow \dfrac{1}{\sqrt{x}}\,dx = dt$

$\int \dfrac{1}{\sqrt{x}}\,dx = \int dt$

$2\sqrt{x} + c_1 = t + c_2$

$2\sqrt{x} = t + c$

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$4(x) = (t+c)^2$

$x = \left(\frac{t+c}{4}\right)^2$

$x' = e^{-2x}$

Solution :-

$x' = e^{-2x}$

$$\frac{dx}{dt} = e^{-2x}$$

$$\frac{dx}{dt} = dt$$

$$\int \frac{1}{e^{-2x}} dx = \int dt$$

$$\frac{e^{2x}}{2} = t+c$$

$$\frac{e^{2x}}{2} = t+c$$

$$e^{2x} = 2(t+c)$$

$$2x = \ln 2(t+c)$$

$$X = \frac{\ln 2(t+c)}{2}$$

$$y' = 1 + y^2$$

Solution:-

$$\frac{dy}{dt} = 1+y^2$$

$$\frac{dy}{1+y^2} = dt$$

$$\int \frac{1}{1+y^2}\,dt = \int dt$$

$$y = \tan(t+c)$$

$$v' = \frac{1}{5-2v}$$

Solution:-

$$\Rightarrow \frac{dy}{dt} = \frac{1}{5-2y}$$

$$\Rightarrow \frac{dy}{5-2v} = dt \quad (1)$$

$$\int \frac{dy}{5-2y} = \int dt$$

$$\Rightarrow -(v-5)v + c_1 = t + c_2$$

$$\Rightarrow -(v-5)v = t+c$$

$$\Rightarrow v(1) = \frac{t+c}{n(v-5)}$$

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$x' = ax + bx, \quad a, b > 0$

Solution:

$$\theta' = \frac{\theta}{4+4\theta^2}$$

$$\frac{d\theta}{dt} = \frac{\theta}{4+\theta^2}$$

$$\frac{\theta^2 d\theta}{\theta} = dt$$

$$\frac{\theta^2}{\theta} = \int dt$$

$$3\ln(\theta) + \frac{\theta^2}{2} + c$$

$x' = e^{x^2}$.

Solution:-

$$\frac{dx}{dt} = e^{x^2}$$

$$\frac{dx}{dt} = dt$$

$$\frac{dx}{e^{x^2}} = dt$$

$$\int \frac{1}{e^{x^2}} du = \int dt$$

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$y' = r(a-y)$

Solution:-

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\frac{1}{r(a-y)}\, dy = \int dt$$

$$\frac{(a-y)u}{r} = t+c$$

$$\Rightarrow u(t) = \frac{r(t+c)}{a-y} \quad A$$

Q2:- Solve $y' = r(a-y)$ when r and a are constants

Solution:-

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)}\, dy = \int dt$$

$$a - y = k(r)^{1/2}$$

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Q3 Solution :-

put $x(0) = 1$

$$x(t) = \frac{(t+c)^2}{4}$$

$x(0) = 1$ $x(t) = x(0) = 1$

$$1 = \frac{(0+c)^2}{4}$$

$$\sqrt{c^2} = \sqrt{4}$$

$$\boxed{c = 2}$$

) Solution :-

$$x(t) = \frac{\ln_2(t)+c}{2}$$

$$x(0) = 1$$

$$1 = \ln_2(0)+c$$

$$1 = \ln + c$$

$$2 = \ln c^2$$

Question 4 (9)

$$x' = \frac{2x}{t+1}$$

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(2-3)

$$\frac{dx}{dt} = \frac{2x}{t+1}$$

$$\frac{dx}{2x} = \frac{1}{t+1}\,dt$$

$$\int \frac{dx}{2x} = \int \frac{1}{t+1}\,dt$$

$$\frac{x^2}{4} = \ln(t+1) + C$$

$$\theta' = t\sqrt{t^2+1}\,\sec\theta$$

Solutions

$$\frac{d\theta}{dt} = t\sqrt{(t^2)+1}\,\sec\theta$$

$$\frac{d\theta}{\sec\theta} = t(t^2+1)\,dt$$

$$\int \frac{d\theta}{d\sec\theta} = \int t\sqrt{t^2+1}\,dt$$

$$\Rightarrow \sin\theta = \frac{(t^2+1)^{3/2}}{3} + C$$

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