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Q1. Compute adjoint of;

(i) $A = \begin{bmatrix} 1 & 2 & 2nd\text{-}ID \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

First of all we find cofactors of "A"

$A_{ij} = (-1)^{i+j} \cdot m_{ij}$

$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = +1(6-1) = \boxed{5}$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = -1(4-3) = \boxed{-1}$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = +1(2-9) = \boxed{-7}$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 6 \\ 1 & 2 \end{vmatrix} = -1(4-6) = \boxed{2}$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix} = 1(2-18) = \boxed{-16}$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -1(1-6) = \boxed{5}$

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$$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 6 \\ 3 & 1 \end{vmatrix} = 1(2-18) = \boxed{-16}$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 6 \\ 2 & 1 \end{vmatrix} = -1(1-12) = \boxed{11}$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1(3-4) = \boxed{-1}$$

Now find adjoint.

$$A = \begin{bmatrix} 5 & -1 & -7 \\ 2 & -16 & 5 \\ -16 & 11 & -1 \end{bmatrix}$$

Tating transpose.

$$A = \begin{bmatrix} 5 & -1 & -7 \\ 2 & -16 & 5 \\ -16 & 11 & -1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 5 & 2 & -16 \\ -1 & -16 & 11 \\ -7 & 5 & -1 \end{bmatrix}$$

This is adjoint of "A".

(ii) $B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$

first of All we find Cofactors.

$A11 = (-1)^{1+1} \begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix} = 1(-8+16) = \boxed{8}$

$A12 = (-1)^{1+2} \begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix} = -1(16-40) = \boxed{24}$

$A13 = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = 1(-4+5) = \boxed{1}$

$A21 = (-1)^{2+1} \begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix} = -1(32+10) = \boxed{-42}$

$A22 = (-1)^{2+2} \begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix} = 1(16-40) = \boxed{-24}$

$A23 = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -1(-6-20) = \boxed{26}$

$A31 = (-1)^{3+1} \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = 1(32+5) = \boxed{37}$

$A32 = (-1)^{3+2} \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = -1(24-10) = \boxed{-14}$

$A33 = (-1)^{3+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = 1(-3-8) = \boxed{-11}$

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Now find adjoint:

$$B = \begin{bmatrix} 8 & 24 & 1 \\ -42 & -24 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

Taking transpose -

$$B^t = \begin{bmatrix} 8 & -42 & 37 \\ 24 & -24 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

This is adjoint of "B".

Q3 $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$

$\mathrm{Det}(A - \lambda I) = 0$ E value

$\mathrm{Det}(A - \lambda I)x = 0$ Vector

$\mathrm{Det}(A - \lambda I) = 0$

$$\left| \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \right|$$

$$\left| \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right|$$

$$\left| \begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{bmatrix} \right|$$

$(2-\lambda)\left[(3-\lambda)(2-\lambda) - 2\right] - 1((2-\lambda)+2) + 1(1+3-\lambda) = 0$

$(2-\lambda)\left[6 - 3\lambda - 2\lambda + \lambda^2 - 2\right] - \left[2 - \lambda + 2\right) + 1(4-\lambda) = 0$

$(2-\lambda)(4 - 5\lambda + \lambda^2) - (4-\lambda) + (4-\lambda) = 0$

$= 8 - 10\lambda + 2\lambda^2 - 4\lambda + 5\lambda^2 - \lambda^3 - \cancel{4} + \cancel{\lambda} + \cancel{4} - \cancel{\lambda} = 0$

$= -\lambda^3 + 7\lambda^2 - 14\lambda + 8 = 0$

$$\begin{array}{r|rrrr} & -1 & 7 & -14 & 8 \\ +2 & \downarrow & -2 & +10 & -8 \\ \hline & -1 & 5 & -4 & 0 \end{array}$$

$-\lambda^2+5\lambda-4=0$

$\lambda^2-5\lambda+4=0$

$\lambda^2-4\lambda-\lambda+4=0$

$\lambda(\lambda-4)-1(\lambda-4)=0$

$(\lambda-1)(\lambda-4)=0$

$\lambda-1=0, \lambda-4=0$

$\lambda=1, \quad \lambda=4$

$\lambda=2, \lambda=1, \lambda=4$

Eigen values $\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$

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For Eigen vector

Det $(A-\lambda I)x=0$

$$\left| \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{vmatrix} 2-2 & 1 & 1 \\ 1 & 3-2 & 2 \\ -1 & 1 & 2-2 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$0x+y+z=0$ — ①

$x+y+2z=0$ — ②

$-x+y+0=0$ — ③

$y+z=0$

$\underline{-x \pm y + 0 = 0}$

$x+z=0$ —(4)

$x=-z$ —(5) put in (2)

$-z+y+2z=0$ Add (6) + (1)

$y-z=0$ —(6)

$\underline{y+z=0}$

$2y=0$

$\boxed{y=0}$

put in (1)

$0+z=0$

$\boxed{z=0}$ —

$x+y+2z=0$

$0+0+0=0$

proved

Eigen vector

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Q21. Find the Cofactors of $A_{21}$, $A_{31}$, $A_{33}$ if

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = (-1)(-4+9) = \boxed{-5}$$

∴ Cofactor of $A_{21}$ is $\boxed{-5}$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = 1(-2-9) = \boxed{-11}$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} = -1(3-4)$$

$$= \boxed{1}$$

Cofactors =

$A_{21} = -5$

$A_{31} = -11$

$A_{33} = 1$ Ans