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ID: 7706

SECTION: "C"

SUBJECT: INTR. TO Structural Dyanamic and Earthquake Engineering

INSTRUCTOR: ENGR. YASEEN MEHMOOD

QUESTION # 01

ANSWER :-

Given Data

Length of beam = l = 10'

Beam pulled in downward = 1/2"

E = 2900 ksi

I = 150 in^4

6_{st} = 7706 lb

Ignore self weight and damping effect

Required:-

1- Natural time period = ?

2- Develop and solve equation of motion for vibration at free end = ?

3- Develop eq. showing variation in the equivalent static force with time = ?

4- Amplitude of equivalent static force = ?

5- Graph = ?

Solution:-

The general equation of Motion for SDOF system is.

$$Ku + C\dot{u} + m\ddot{u} = P(t)$$

Undamped system ($c=0$) undergoing free vibration ($p(t)$) = 0

so general EOM because

$$ku + m\ddot{u} = 0 \quad \text{--- (i)}$$

$$k = \frac{3EI}{l^3}$$

$$k = \frac{3 \times 2900 \times 150}{(10 \times 12)^3}$$

$$\boxed{k = 7.55 \text{ k/in}}$$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental with like lb, ft or kg, m, sec

$$k = 7.55 \text{ k/in} = 90625 \text{ lb/ft}$$

$$m = \frac{7706 \text{ lb sec}^2}{32.2 \text{ ft}}$$

$$\boxed{m = 239.31 \text{ slug}}$$

$$w_n = \sqrt{k/m} = \sqrt{\frac{96025}{239.31}}$$

$$\boxed{w_n = 20.03 \text{ rad/sec}}$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{20.03}$$

$$\boxed{T_n = 0.313 \text{ sec}}$$

Putting values in eq-(i)

$$90625\,\ddot{u} + 239.31\,u = 0$$

where k is in lb/ft and m is in lb sec/ft²

General solution to the EOM for undamped free vibration is,

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$$u(0) = \frac{1}{2}'' = \frac{1}{24}\text{ ft} \quad \text{and} \quad \dot{u}(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(20.03\,t) + 0$$

$$\boxed{u(t) = \frac{1}{24} \times \cos(20.03t)}$$

Equivalent static force at any time "t" is,

$$f_s(t) = k \cdot u(t) =$$

$$= \frac{90625 \times \cos(20.03t)}{24}$$

$$f_s(t) = 3776 \cos(20.03\,t)$$

Amplitude of dynamic displacement, u_o for undamped free vibration is

$$u_o = \sqrt{\left[(u\cos)^2 + \left(\frac{u(o)}{\omega_4}\right)^2\right]}$$

$$= \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

$$u_o = 1/24 \text{ ft}$$

Amplitude of equivalent static force,

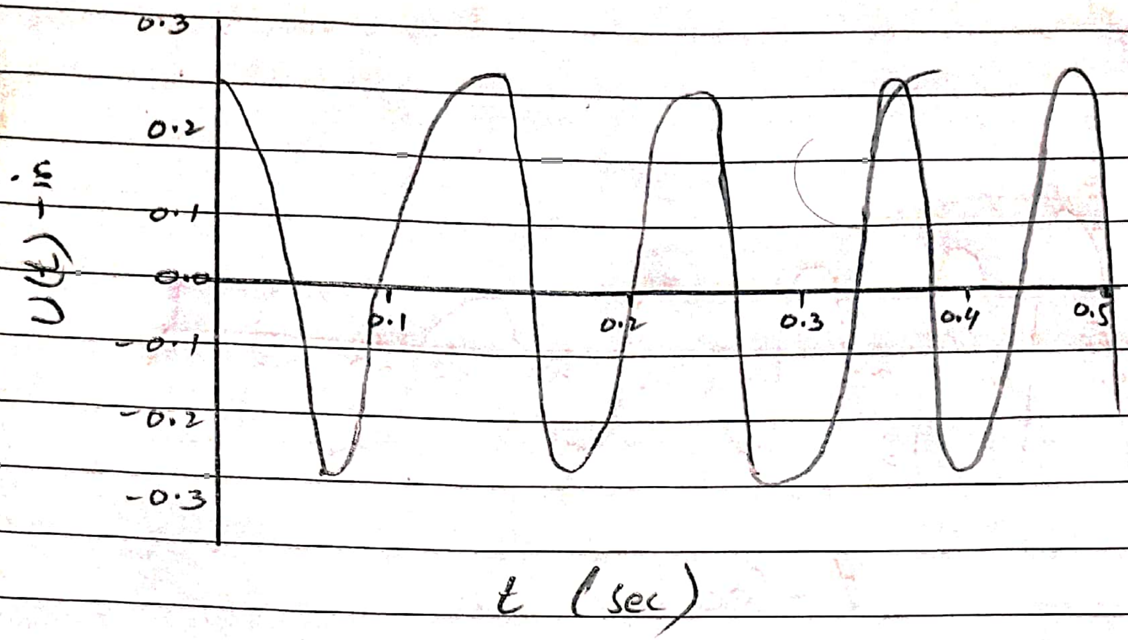
$$Ku_o = 90625 \times 1/24$$

$$Ku_o = 3776 \text{ lb}$$

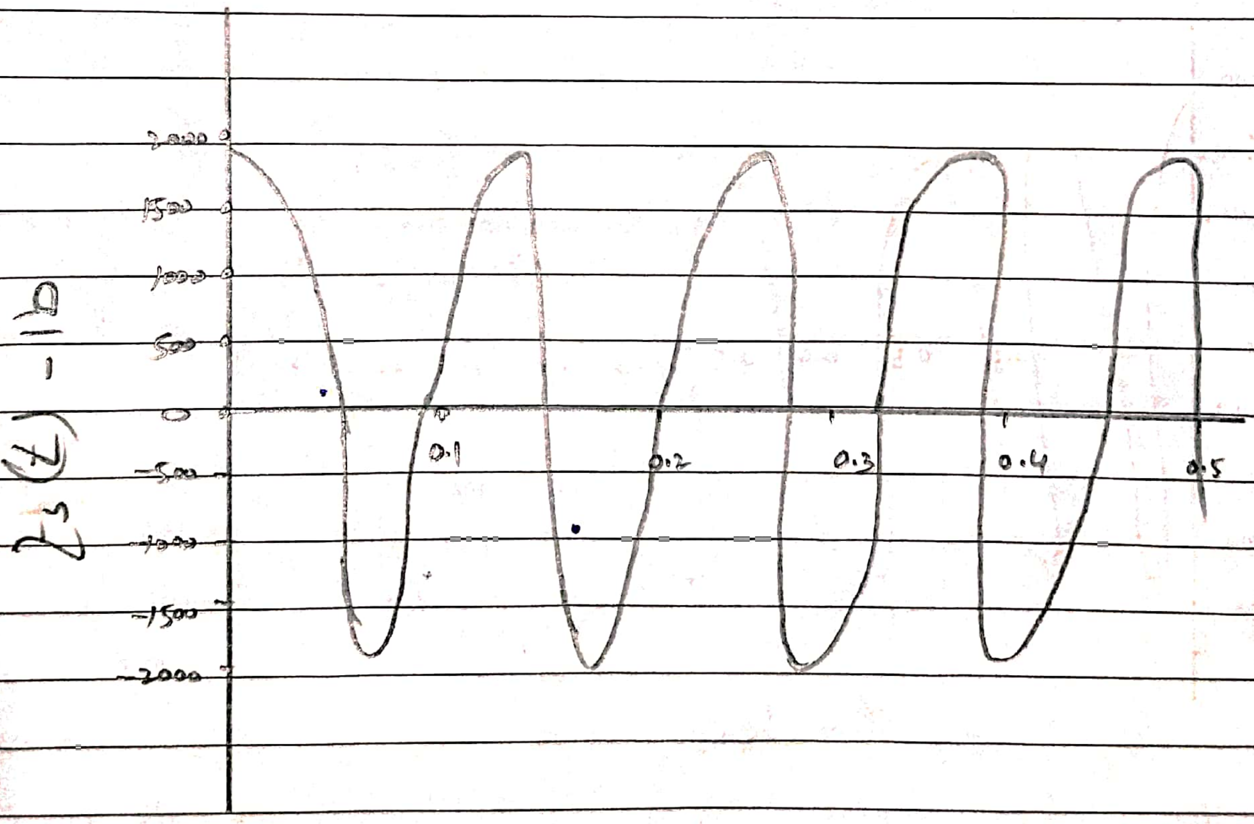
5A

Date:

UNDAMPED FREE VIBRATION :-



Variation of displacement with time



QUESTION #02

ANSWER :

Given Data:

From Question "01"

Required:

1- Develop and solve eq of motion for vibrations resulting at free end.

2- Develop equations showing vibration in equivalent static forces with time.

3. Draw graph to show the variation of displacement with time and vibration of equivalent static forces with time.

Solution :-

Damping Ratio for RCC

Min = 0.80% , Avg = 1.3%

So,

we consider 1.3%

Equation of Motion for damped free vibration

$k u + c \dot{u} + m \ddot{u} = 0$ —— (i)

From problem (1)

$k = 90625$ lb/ft

$m = 239.31$ lb. sec^2/ft

$c = \xi \times 2 m \omega_n$

~~c = 2 × 240.~~

$c = 2 \times 239.31 \times 20.03 \times 0.013$

$$c = 124.62 \text{ lb.sec/ft}$$

putting values of k, m, and c in eq (i)

(i) ⇒ $$90625 u + 124.62 \dot{u} + 239.31 \ddot{u} = 0$$

Solution to the EOM for damped free vibration

$$u(t) = e^{-\xi \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\,\xi\omega_n\right]\sin(\omega_D t)\right]$$

$\omega_D = 20.03$ rad/sec

$$u(t) = e^{-0.013t}\left[\frac{1}{24} \times \cos(20.03t) + \frac{1}{20}\left[0 + \frac{1}{24} \times 0.013 \times 20.03 \sin(20t)\right]\right]$$

$$\boxed{u(t) = e^{-0.26t}\left[0.041 \times \cos(20.03t) + 0.0005416 \sin(20.03t)\right]}$$

~~$u(t) = e^{-0.26t}[$~~

$$f_s(t) = K \cdot u(t) = 90625 \times u(t)$$

$$\boxed{f_s(t) = e^{0.26t}\left[3715 \cos(20.03t) + 49.08 \sin(20.03t)\right]}$$

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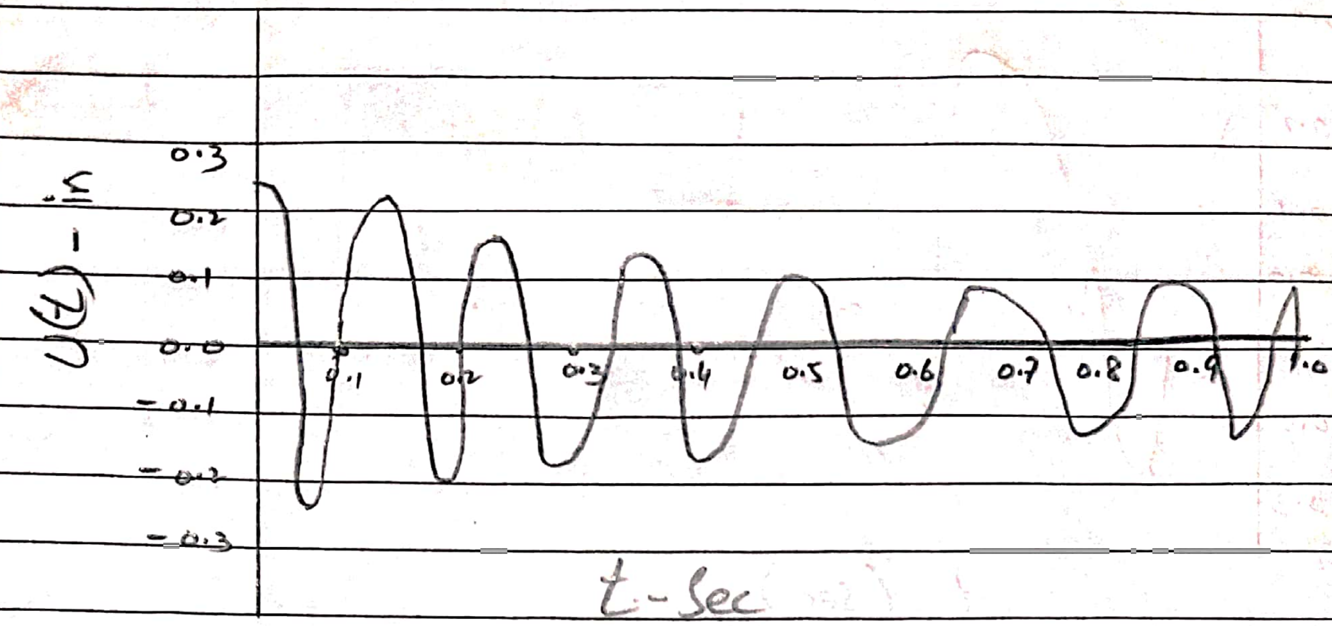
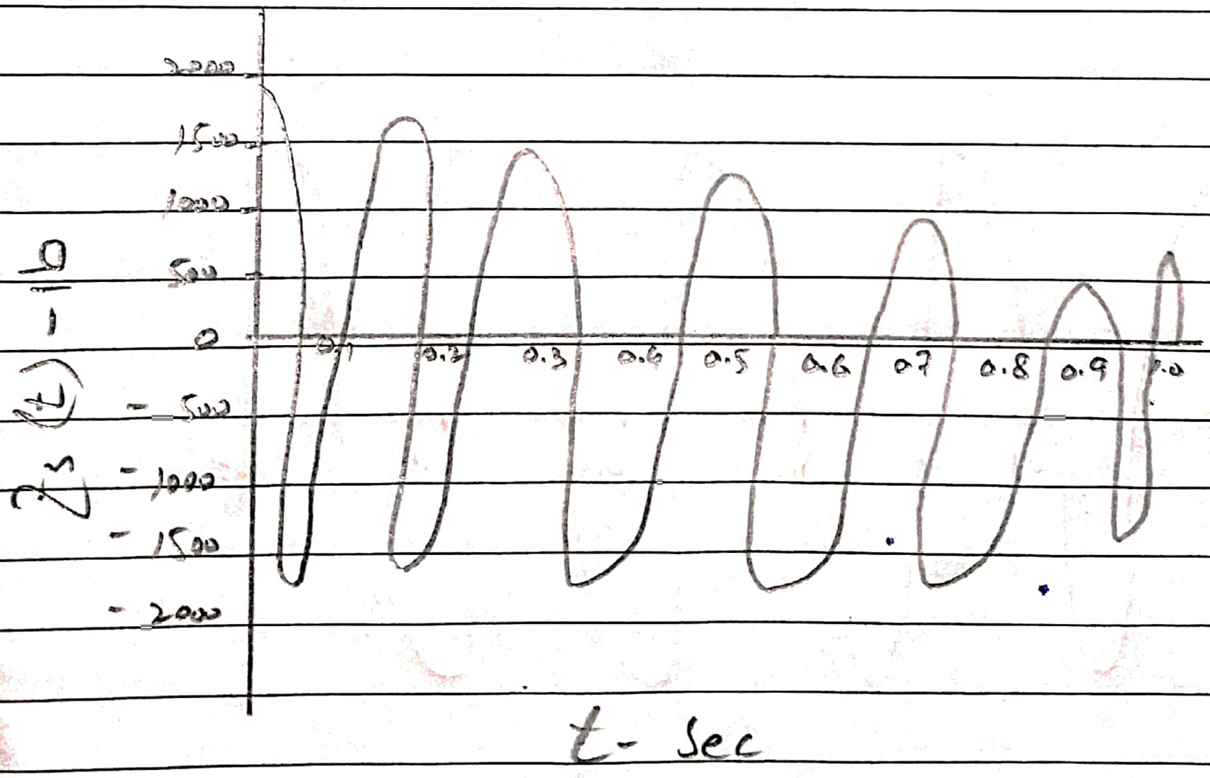
⇒ DAMPED FREE VIBRATION



⇒ DAMPED FREE VIBRATION



QUESTION #03

ANSWER

Given Data

Force = F = 60 kips

Displacement = 7700/1000 = 7.70"

No. of cycle 7 are complete in 3.57 sec

Amlitude = 2.286 cm = 0.9"

Ignore vertical vibration.

Required.

1. Damping ratios
2. Natural period of un-damped vibration
3. Stiffness of structures
4. Weight of tank
5. Damping coefficient
6. No of cycles to reduce displacement to 0.5"

Solution.

$u_1 = 7.70"$

After $j = 7$, $u_{j+1} = u_8 = 0.9"$

1. ζ = Damping ratio = ?

$$\zeta = \frac{1}{2\pi 3} \ln\left[\frac{u_i}{u_{i+1}}\right]$$

$$7 = \frac{1}{2\pi\beta} \ln\left[7.70/0.9\right]$$

$$\beta = \frac{0.150}{2\pi(7)}$$

$$\beta = 0.0488$$

$$\boxed{\beta = 4.88\%}$$

2- $T_n = ?$

7 cycles of vibration are completed in 3.57 secs.

Now,

Time required for 1 cycle = T_o = $\frac{3.57}{7}$

$T_o = 0.51$ sec

Now

$$w_o = w_n \sqrt{(1-\beta^2)}$$

$$\frac{2\pi}{w_o} = \frac{2\pi}{w_n\sqrt{1-\beta^2}}$$

$$T_o = \frac{T_n}{\sqrt{1-\beta^2}}$$

$T_n = T_n \times \sqrt{1-\beta^2}$

$T_n = 0.51 \times \sqrt{1-(0.0438)^2}$

$T_n = 0.5093$

$$\boxed{T_n = 0.51 \text{ sec}}$$

③ $K = ?$

$K = \frac{60 \times \cos 60^\circ}{2} = 15 \; K/in$

$$\boxed{K = 18000 \; lb/ft}$$

4- Weight of tank $W = ?$

$W_n = \sqrt{K/m} = \sqrt{\frac{K}{W/g}} = \sqrt{\frac{K \times g}{W}}$

$W_n^2 = \frac{K \times g}{W}$

$W = \frac{K \times g}{W_n^2}$

Also $W_n = \frac{2\pi}{T_n}$

$W = \frac{K \times g}{\frac{4\pi^2}{T_n^2}}$

$$W = K \times g \times \frac{T_n^2}{4\pi^2}$$

$$W = 18000 \times 32.2 \times \frac{(0.51)^2}{4\pi^2}$$

$$W = 3818.64 \text{ lb}$$

$$\boxed{W = 3.81 \text{ K}}$$

5- $C = ?$

it is known that $\xi = \frac{C}{2m\omega_n}$

$$C = \xi = 2m\omega_n$$

$$C = \xi \times 2m\left(\frac{2\pi}{T_n}\right)$$

$$C = 0.0488 \times 2 \times \left(\frac{3818.64}{32.2}\right)\left(\frac{2\pi}{0.51}\right)$$

$$\boxed{C = 142.59 \text{ lb-sec/ft}}$$

6- No. of cycle to reduce displacement amplitude from 770' to 0.5, $j = ?$

$$j = \frac{1}{2\pi\xi} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

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$$j = \frac{1}{2\pi \times 0.0488} \ln\left[\frac{7.70}{0.5}\right]$$

$$j = \frac{1}{2\pi \times 0.0488} (2.7343)$$

$$j = 8.91 \text{ or } 9$$

cycles