Question : 1

Answer the following:

a) Mean and Variance of binomial distribution is 4 and Find n and p.

Given:

Variance $= \sigma x^2 = 4$

Mean $= \mu x = 4$

Find:

$n = ?$

$p = ?$

Solution:

$\mu = np = 4$ —— (1)

$\sigma^2 = npq = 4$ —— (2)

Dividing (2) by (1)

$$\frac{\cancel{np}q}{\cancel{np}} = \frac{\cancel{4}}{\cancel{4}}$$

$$\boxed{q = 1}$$

As,

$p + q = 1$

$p = 1 - q$

$p = 1 - 1$

$$\boxed{p = 0}$$

Now,

$np = 4$ by eq (1)

$n = \frac{4}{p}$

$n = \frac{4}{0}$

$$\boxed{n = \infty}$$

Hence p is 0 and q is ∞

b) If X is normally distributed with mean 12 and standard deviation 4 then find the probability if: ②

Given :-

Mean = 12

Standard deviation = 4

Find:

Probability = ?

Solution:

Mean $= \mu = np = 12$ —— ①

SD $= 6 = \sqrt{npq} = 4$ —— ②

Dividing ① and ②

$$\frac{\mu}{6} = \frac{np}{\sqrt{npq}} = \frac{\cancel{12}^{3}}{\cancel{4}}$$

$$\frac{np}{\sqrt{npq}} = 3$$

Squaring both sides

$$\frac{(np)^2}{npq} = (3)^2$$

$$\frac{n^2 p^2}{\cancel{n}\cancel{p}q} = 9$$

$$\frac{np}{q} = 9$$

$$np = 9q \quad \text{—— ③}$$

Now

$$np = 12 \quad \text{—— ④}$$

Subtracting ③ and ①

$$-np = 9q$$

$$\oplus np = \oplus 12$$

$$0 = 9q - 12$$

$$9q - 12 = 0$$

$$q = \frac{\cancel{12}\,4}{9\,3}$$

$$q = \frac{4}{3} > 1$$

The statement is incorrect as q can not be greater then 1.

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c) Define critical region.

Critical Region:-

A critical region, also known as the rejection region, in a set of values for the test statistics for which the null hypothesis is rejected. i-e if the observed test statistics is in the critical region then we reject the null hypothesis and accept the alternative hypothesis.

Example:

The critical values for a 5% significance test are: For a one tailed test, the critical value is 1.645 so the critical region is $Z < -1.645$ for a left tailed test and $Z > 1.645$ for a right tailed test.

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d) Write properties of t-distribution.

Properties of t-distribution:

The t distribution has the following properties. The mean of the distribution is equal to 0. The variance is equal to $v/(v-2)$, where v is the degrees of freedom and $v > 2$. The variance is always greater than 1, although it is close to 1 when there are many degrees of freedom.

(i) The distribution has mean 0.

(ii) The distribution is symmetric about the mean.

(iii) The variance is equal to $\frac{df}{df-2}$

(iv) The student distribution ranges from $-\infty$ to ∞ (infinity).

e) Write a short note on analysis of variance.

Analysis of Variance :-

Analysis of variance is a collection of statistical models and their associated estimation procedures used to analyze the differences among group mean in a sample. ANOVA was developed by the statistician Ronald Fisher. The ANOVA is based on law of total variance, where the observed variance in a particular variable is partitioned into components attributed to different sources of variation.

Characteristics:

ANOVA is used in analysis of comparative expeiments, those in which only the difference in outcomes is of interest. The statistical significance of the expeiment is determined by a ratio of two variance.

Advantages:

(i) Simplest method to test the treatment effects as well as block effects.

(ii) Statistical analysis also simple because it is based on two-way classification.

(iii) More efficent than CRD.

(iv) Trend effect is reduced.

9) Define statistical quality control.

Statistical quality Control:-

Statistical quality control is the term used to describe the set of statistical tools used by quality professionals.

The use of statistical quality control methods in the monitoring and maintaining of the quality of products and services.

Statistical quality Control Categories:-

(i) Descriptive Statistics.

ii) Statistical process Control

h) Define the term "Chance causes and assignable causes

Chance Causes :-

A process that is operating with only chance causes of variation present is said to be in statistical control. In other words, the chance causes are an inherent part of the process.

(i) Consists of many individual causes.

ii) Slight variation in raw material

iii) Lack of human variation and perfection in reading instruments.

Assignable Causes :-

Assignable causes is an indentifical specific cause of variations in a given process or measurement.

(i) Consists of one or just a few individuals causes.

ii) Any one assignable cause can result in a large amount of variation.

iii) Batch of raw material.

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i) Define traffic intensity.

Traffic intensity :-

A measure of the average occupancy of a facility during a specified period of time, normally a busy hour, measured in traffic units and defined as the ratio of the time during which a facility is occupied to the time this facility is available for occupancy.

In a digital network, the traffic intensity is

$$\frac{aL}{R}$$

where

a is the average arrival rate of packets.

L is average packet length.

R is the transmission rate.

j) Write the characteristics of queuing theory.

Characteristics of Queuing Theory:-

A queuing system is specified completely by the following five basic characteristics. The input process. It expresses the mode of arrival of customers at the service facility governed by some probability law. The number of customers emanate from finite or infinite sources

(i) The arrival pattern.

(ii) The service mechanism.

(iii) The queue discipline

(iv) The number of customers allowed in system.

(v) The number of service channels.

Question : 02

a) Derive mean and variance of binomial distribution.

Solution:-

The binomial distibution

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

for $x = 0, 1, 2, \ldots n$

$$\mu = np \quad , \quad \sigma^2 = np(1-p)$$

$$\downarrow \qquad\qquad \downarrow$$

$$E(x) \qquad\qquad Var(X)$$

A burnoli Random variable can be thaught of as sum of n independent random variables, each with p mean variance $p(1-p)$

Let $V_1 \ldots V_n$ be independent Bernoli variables (8)

$E(U) = p$, $Var(V_i) = p(1-p)$

$X = U_1 + \cdots + U_n$

$E(X) = E(U_1 + \cdots + U_n)$

$E(X) = E(U_1) + \cdots E(U_n)$

$E(U_i) = p \quad Var(V_i) = p(1-p)$

$X = U_1 + \cdots + U_n$

$Var(X) = Var(U_1 + \cdots + U_n)$

$Var(X) = Var(U_1) + \cdots + Var(U_n)$

$= p(1-p) + \cdots + p(1-p)$

$= np(1-p)$

The binomial therom

$E(X) = \Sigma x p(x)$

$$= \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,(n-1)! - (x-1)!} \times$$

$$p^{x-1} (1-p)^{(n-1)(x-1)}$$

when $m = n-1$

then $y = x-1$

$$= np \sum_{y=0}^{m} \frac{m!}{y!\,(m-y!)} p^y (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}$$

$$E(x) = np(p + (1) - p))^m = np(1)$$

$$\boxed{E(x) = np} \rightarrow \text{Mean}$$

$$Var(x) = E\left[(X-\mu)^2\right]$$

$$= \sum_x (x-\mu)^2 \, p(x)$$

$$E(X^2) = \sum_{x=0}^{n} x^2 \frac{n!}{X!\,(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(n-2)!,\,(n-x)!} p^{x-2} \times (1-p)^{n-x}$$

$$E[X(X-1)] = n(n-1)p^2 \times \sum_{x=2}^{n} \frac{(n-2)!}{(X-2)!\,(n-2)-(x-2)} p^{x-2} (1-p)^{(n-2)(x-2)}$$

$$= n(n-1)p^2 \times \sum_{y=0}^{m} \frac{m!}{y!,\,(m-y)!} p^y (1-p)^{m-y}$$

$$= (p + (1-p))^m = 1^m = 1$$

$$E(x(x-1)) = En(n-1)p^2$$

$$E(X^2 - 1) = n(n-1)p^2$$

$$E(x)^2 - E(x) = n(n-1)p^2$$

$$\boxed{E(x)^2 = n(n-1)p^2} \rightarrow \text{Variance}$$

Question = 2 part (B)

Solution :-

Let x denote no. of cars which are hired out per day.

Given = Poisson mean = $m = 1.5$

Now

$$P(X=x) = \frac{e^{-m} \cdot m^x}{x!} = \frac{e^{-1.5} \cdot 1.5^x}{x!}$$

1) P (neither car used)

$$P(X=0) = \frac{e^{-1.5} \cdot 1.5^0}{0.2231}$$

2) P (some demand refused) = P(demand is more than 2 cars per day

$$P(x>2)$$

$$= 1 - P(x \le 2)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[\frac{e^{-1.5}\ 1.5^0}{0!} + \frac{e^{-1.5}\ 1.5}{1!} + \frac{e^{1.5}\ 1.5^2}{2!}\right]$$

$$= 1 - e^{1.5}\left[1 + 1.5 + \frac{2.25}{2}\right]$$

$$= 0.1912$$

Now, proportion of days on which neither car used = 0.2231 = 22.31%.

proportion of days on which some demand refused = 0.1912 = 19.12%.

Question = 3

A set of 5 assembles of 15 sub-groups.

Solution:-

Range : (40 — 93)

smallest value = 40

largest value = 95

For 5 assembles chart is given.

Group no.	Range of defects	frequency
1	40 - 50	4
2	51 — 60	3
3	61 - 70	2
4	71 - 80	3
5	81 - 95	3

The maximum frequency has deflects b/w 71 – 95.

The group 4 and 5 have maximum no. of defects respectively as shown in chart above.