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Paper: Calculus Engineering
Section: A

Q1 Find PQ where P is the point in three dimensional space with coordinates (4, 1, 3) and the point Q with coordinates (1, 2, 4). Find the distance between P and Q. Further, find the position vector of the point dividing PQ in the ratio 1:3

Sol^n Coordinate of P = (4, 1, 3)

$$OP = 4i + 1j + 3k$$

or $OQ = (1, 2, 4)$

$$= 1i + 2j + 4k$$

$$\vec{OQ} - \vec{OP} = (i + 2j + 4k) - (4i + 1j + 3k)$$

$$= -3i + 1j + 1k \rightarrow (1)$$

Now distance between P and Q = |PQ|

$$|PQ| = \sqrt{(-3)^2 + 1^2 + 1^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$|PQ| = \sqrt{11} \rightarrow (2)$$

let M be the point which divided PQ in ratio 1:3, then by ratio theorem

Position of vector of M = $\overrightarrow{OM}$

$$= \frac{3(4i+1j+3k)+1(i+2j+4k)}{1+3}$$

$$= \frac{12i+3j+9k+i+2j+4k}{4}$$

$$= \frac{13i+5j+13k}{4} \rightarrow (3)$$

Hence eq (1), (2) and (3) are the required set.

Q2 $$\int \frac{4x^3+10x+4}{2x^2+x}\,dx$$

Sol^n :- Convert into proper

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & 4x^3+10x+4 \\ & \underline{-4x^3 \quad\quad -2x^2} \\ & -2x^2+10x+4 \\ & \underline{+2x^2 + x} \\ & 11x+4 \end{array}$$

$$\Rightarrow \int (2x-1)\,dx + \int \frac{11x+4}{2x^2+x}\,dx \rightarrow (i)$$

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$$\int \frac{11+4}{2x^2+x}\,dx = \int \frac{A}{x}\,dx + \frac{B}{2x+1}\,dx \rightarrow \text{(i)}$$

By Partial fraction.

$$11x+4 = (2x+1)A + xB.$$

$$2x+1=0 \quad ; \quad x=0$$

$$\Rightarrow x = -11,$$

$$x=0 \Rightarrow 11(0)+4 = (2(0)+1)A + 0$$

$$4 = A$$

$$x = -1/2 \Rightarrow 11(-1/2)+4 = \left(2\left(\frac{-1}{2}\right)+1\right)A + -\frac{1}{2}B$$

$$\frac{-11+8}{\cancel{2}} = 0 - \frac{1}{\cancel{2}}B$$

$$-3 = -B$$

$$\boxed{B=3}$$

$$\text{(ii)} \Rightarrow \int \frac{11x+4}{2x^2+x}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\ln|x| + \frac{3}{2}\int \frac{2}{2x+1}\,dx$$

$$= 4\ln|x| + \frac{3}{2}\ln|2x+1| + C$$

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Putting this value in (i).

$$\Rightarrow \int (2x-1)\,dx + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C$$

$$\Rightarrow \frac{2x^2}{2} - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C$$

$$\Rightarrow x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C$$

Q3 Evaluate

(a) $\int_0^2 x^2 e^x \, dx$

Solution:- Integration by parts

$$\int uv = uv - \int u'v$$

$u = x^2$ $\quad\quad v' = e^x$

$u' = 2x$ $\quad\quad v = e^x$

$$= x^2 e^x - \int 2xe^x \, dx$$

$$= x^2 e^x - 2\int xe^x \, dx$$

Now solving xe^x

Integration by parts

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$U = x \qquad V' = e^x$

$U' = 1 \qquad V = e^x$

$= x^2 e^x - 2\left[xe^x - \int e^x\,dx\right]$

$= x^2 e^x - 2\,(xe^x - e^x)\Big|_0^2$

$= x^2 e^x - 2xe^x + 2e^x \Big|_0^2$

$= \left((2)^2 e^2 - 2(2)\,e^2 + 2e^2\right) - (2e^0)$

$= 4e^2 - 4e^2 + 2e^2 - 2$

$\boxed{= 2e^2 - 2}$ ~~Ans~~

Q3(b) $\displaystyle\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Solⁿ Substitute $u = \sqrt{x}$ $\qquad \dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$

So $dx = 2\sqrt{x}\,du$

$= \displaystyle\int_1^2 2\sin u\,du$

$= 2\displaystyle\int_1^2 \sin u\,du$

$= -2\cos u\Big|_1^2$

$u = \sqrt{x}$

$= -2\cos\sqrt{x}\Big|_1^2$

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$= -2\cos\sqrt{2} + 2\cos\sqrt{1}$

$\boxed{= 2[\cos\sqrt{1} - \cos\sqrt{2}]}$ Ans

Q4 $U(x,y,z) = \dfrac{1}{x^2+y^2+z^2}$

Solution:

Laplace eq.

$$U_{xx} + U_{yy} + U_{zz} = 0$$

first fing U_{xx}

$$U_x = \frac{\partial}{\partial x}\left[\frac{1}{\sqrt{x^2+y^2+z^2}}\right]$$

using quotien rule

$$U_x = \frac{\frac{d}{dx}(1)\cdot\sqrt{x^2+y^2+z^2} - \frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2}}{\left(\sqrt{x^2+y^2+z^2}\right)^2}$$

$$U_x = \frac{-\frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2}}{\left(\sqrt{x^2+y^2+z^2}\right)^2}$$

$$U_x = \frac{-\frac{\partial}{\partial x}(x^2+y^2+z^2)}{-\left(\frac{1}{2}-1\right)}$$

$$x^2+y^2+z^2 \quad 2 = \sqrt{x^2+y^2+z^2}$$

$$U_x = \frac{-\cancel{2}x}{x^2+y^2+z^2 \; \cancel{2} \cdot \sqrt{x^2+y^2+z^2}}$$

$$U_x = \frac{-x}{(x^2+y^2+z^2)^{3/2}}$$

$$U_{xx} = \frac{\partial}{\partial x}\left[\frac{-x}{(x^2+y^2+z^2)^{3/2}}\right]$$

use quotient rule

$$U_{xx} = \frac{-(x^2+y^2+z^2)^{3/2} - \frac{\partial}{\partial x}(x^2+y^2+z^2)^{3/2} \cdot x}{(x^2+y^2+z^2)^3}$$

$$U_{xx} = \frac{-(x^2+y^2+z^2)^{3/2} - \frac{3}{2}(x^2+y^2+z^2)^{\frac{3}{2}-1} \cdot \frac{\partial}{\partial x}(x^2+y^2+z^2)\,x}{(x^2+y^2+z^2)^3}$$

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$$U_{xx} = \frac{-(x^2+y^2+z^2)^{3/2} - \frac{3}{2} \cdot x^2+y^2+z^2 \left(-2x^2\right)}{(x^2+y^2+z^2)^3}$$

so $$U_{xx} = \frac{-(x^2+y^2+z^2)^{3/2} - 3 \cdot \sqrt{x^2+y^2+z^2} \cdot x^2}{(x^2+y^2+z^2)^3}$$

Now solving for

$U_{yy} =$

first $$U_x = \frac{\partial}{\partial y}\left[\frac{1}{x^2+y^2+z^2}\right]$$

using quotient rule

$$U_y = \frac{-\partial/\partial y\left(\sqrt{x^2+y^2+z^2}\right)}{\sqrt{x^2+y^2+z^2}^{\,2}}$$

$$U_y = \frac{\frac{\partial}{\partial y}(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{-(\frac{1}{2}-1)} \cdot 2\sqrt{(x^2+y^2+z^2)}^{\,2}}$$

$$U_y = \frac{-y}{(x^2+y^2+z^2)^{3/2}}$$

Now U_{yy}

$$U_{yy} = \frac{\partial}{\partial y}\left(\frac{-y}{(x^2+y^2+z^2)^{3/2}}\right)$$

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$$U_{yy} = \frac{(x^2+y^2+z^2)^{3/2} - \partial/\partial y\,(x^2+y^2+z^2)^{3/2}.y}{(x^2+y^2+z^2)^3}$$

$$U_{yy} = \frac{-(x^2+y^2+z^2)^{3/2} - \frac{3}{\cancel{2}}\sqrt{x^2+y^2+z^2}\,(\cancel{2}y^2)}{(x^2+y^2+z^2)^3}$$

$$U_{yy} = \frac{-(x^2+y^2+z^2)^{3/2}\ 3\sqrt{x^2+y^2+z^2}\ .y}{\cancel{x^2+y^2+z^2}}$$

Now U_{zz}

$$U_z = \frac{-z}{(x^2+y^2+z^2)^{3/2}}$$

$$U_{zz} = \frac{-(x^2+y^2+z^2)^{3/2} - 3\sqrt{x^2+y^2+z^2}\,.z^2}{(x^2+y^2+z^2)^3}$$

$U_{xx} + U_{xx} + U_{zz} = 0$

if it is satisfid then it is laplace equation

$$= \frac{(x^2+y^2+z^2)^{3/2} - 3\sqrt{x^2+y^2+z^2}\,(x^2)}{(x^2+y^2+z^2)^3} + \frac{\left(\begin{matrix}-x^2+y^2+z^2)^{3/2} \\ -3\ x^2+y^2+z^2(y)^2\end{matrix}\right)}{x^2+y^2+z^2} +$$

$$- \frac{(x^2+y^2+z^2)^3 - 3\sqrt{x^2y^2+z^2}}{(x^2+y^2+z^2)^3}$$

$$= \frac{-(x^2+y^2+z^2)^{3/3} + 3\ x^2+y^2+z^2(x^2) - (x^2+y^2+z^2)^{3/2} + 3\sqrt{x^2+y^2+z^2}(y)^2 - (x^2+y^2+z^2)^{3/2} + 3\sqrt{x^2+y^2+z^2}\ (z)^2}{(x^2+y^2+z^2)^3}$$

$$= \frac{-3(x^2+y^2+z^2)^{3/2} + 3\sqrt{x^2+y^2+z^2}\,(x^2+y^2+z^2)}{(x^2+y^2+z^2)^3}$$

$$= \frac{-3(x^2+y^2+z^2)^{3/2} + 3(x^2+y^2+z^2)^{1/2}\cdot(x^2+y^2+z^2)}{(x^2+y^2+z^2)^3}$$

$$= \frac{-3(x^2+y^2+z^2) + 3(x^2+y^2+z^2)^{3/2}}{(x^2+y^2+z^2)}$$

$$= 0$$

$U_{xx} + U_{yy} + U_{zz} = 0$ Hence proved

it is laplace equation