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Question no 1 =

a part

Concider the following analog signals-

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

i) Determine the minimum sampling rate required to avoid aliasing.

Solution:

$f_s \geq 2 F_{max}$ $f = \frac{W}{2\pi}$

$f_1 = \frac{100\pi}{2\pi}$ $f_2 = \frac{200\pi}{2\pi}$

$f_1 = 50Hz$. $f_2 = 100Hz$

so f_2 is max (greater Than f_1)

$f_s \geq 2\times100\ Hz$. sample frequency to avoid aliasing.

(ii) Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after samling? Also explain the effect of this sampling rate onthe newly generated discrete time signal.

Solution:

$F_s = 100\ Hz$

f_1 becomes.

$f_1 = \frac{f_1}{F_s} = \frac{1\cancel{0}}{1\cancel{00}} = 0.5\ Hz$

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f_2 becomes

$$f_2' = \frac{f_2}{100} = \frac{100}{100} = 1Hz.$$

so $w_1 = 2\pi f_1'$ $w_2 = 2\pi f_2'$

$w_1 = 2\pi \times 0.5$ $w_2 = 2\pi \times 1$

$w_1 = \pi$ $w_2 = 2\pi.$

$x[n] = 3\cos 100\pi n + 4\sin 200\pi n$

The signal are.

$x[n] = 3\cos \pi n + 4\sin 2\pi n.$

The effect of sampling rate on the newly generated descrete time signals is that There will be no alising phenomenam means There will not present unwanted compoent in The reconstruction of the signals. The reconstruct orignal signals.

$W_1 = 100\pi$ $f_d = 200\pi$

$f_1 = \frac{\cancel{100}}{\cancel{2\pi}}$ $f_d = 100\pi$

$\boxed{f_1 = 50}$

(iii) What is the analog signal $y_a(t)$ we can reconstouct from the samples if we use ideal interpolation?

Solution:

- Folding frequency of the sampled signal is

folding frequency $= F_s/2 \Rightarrow \frac{100}{2}.$

$= 50Hz$

- We have frequency of The orignal signals

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$f_1 = 50Hz$, $fr = 100Hz$

Both. The frequency are either equal or greater.

The Folding frequency

Hence for ideal interpolation we on construct The orignal signal.

$$x_a(t) = 3\cos 100\pi \; 4 \sin 200\pi t.$$

The orignal signal is constructed we use sampling frequency at Nyeusist rate.

we can also reconstruct The signal for sampling frequency above The nyesist rate.

b part

Consider a discrete time signal which is given by

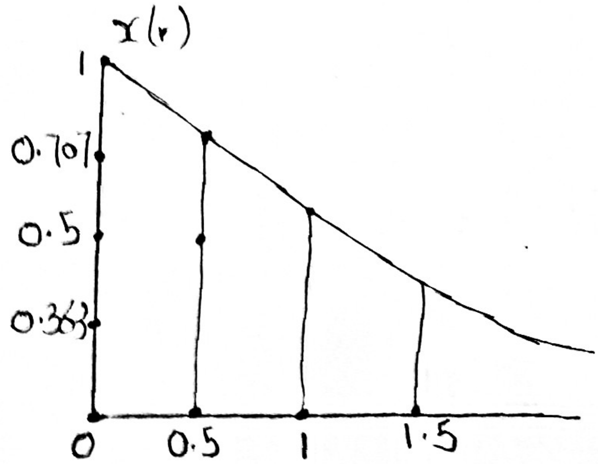
$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

This is signal is sampled at the rate $F_s = 2Hz$.

(i) Draw the sampled signal.

Solution:

x_n	$= 0.5^n$
0	1
0.5	0.707
1	0.5
1.5	0.353



T = 0.5 sec

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(ii) The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i.

Solution:

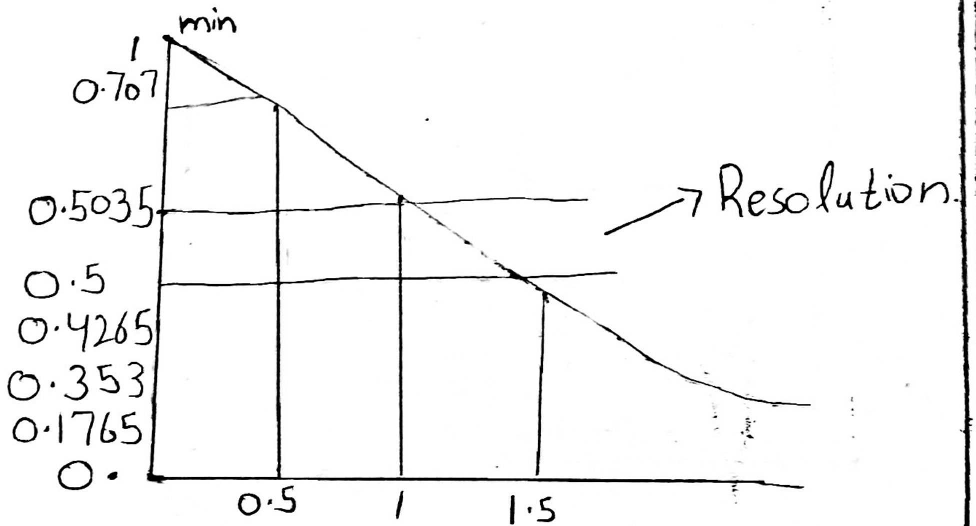
$$L = 2^n$$

$$n = \text{bits} = 3$$

$$L = 2^3 = 8 \text{ levels}$$

$$\text{Resolution} = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1-0}{8} = 0.125$$



(iii) Perfom the process of instruction and rounding off on all the values of the sampled signal and find the quantiztion error for each of the sampled data.
Express your answer in tabular form.

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Solution:

	Discret time signal	In	Reading	errors
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6635	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

Question no 2:

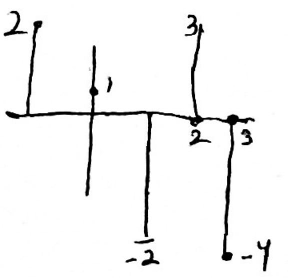
a part

Determine the response of the system to the following input signal with the given impulse response.

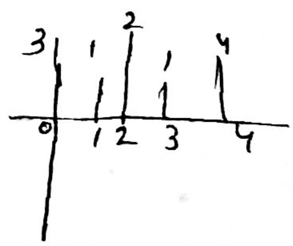
$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\} , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

Solution:

$$Y[n] = \sum_{1000}^{\infty} x[k]h(n-k)$$



n(k)



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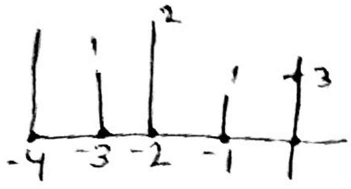
h(-k) folded signal.



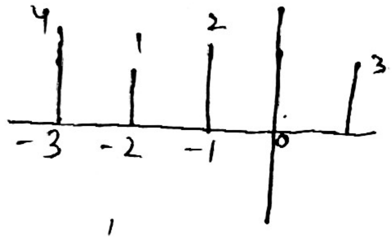
$Y[0] = \sum_{k=1} x(-1)h(-1) + x(0)h(0)$

$Y(0) = (2)(1) + (1)(3)$

$= 2 + 3 = 5$

for n = 1

h(1-k)



$Y(1) = \sum_{x=-1}^{1} x(A)\, n(1-k)$

$= x(-1)h(-1) + x(0)h(0) + 0x(1)h(1)$.

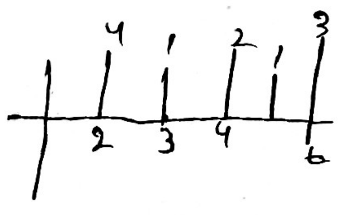
$+ (x)\ h(1) + x(2)n(2) + 0(x(3)n(3))$

$Y(2) = (2)(4) + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$

$= 8 + 1 - 4 + 3 - 12 = -4$

n = 3

n(3-x)



$Y(3) = \sum_{k=2} x(x)\, h(n-k)$

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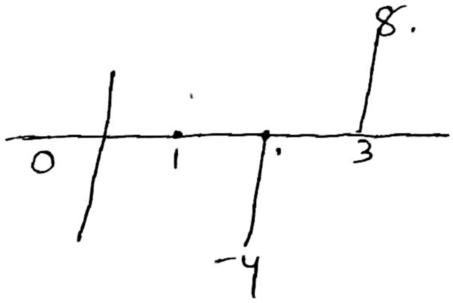
$$= x(2)h(2) + x(3)h(3)$$

$$(3)(4) + (-4)(1) = 12 - 4 = 8.$$



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Question no : 2.

b part

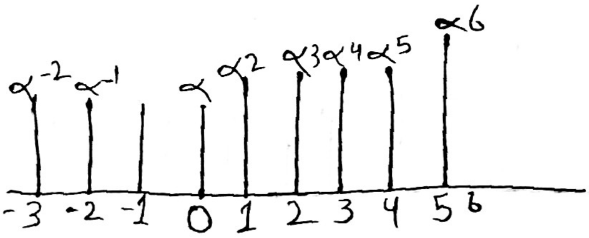
Compute the convolution $y(n)$ of the following signal.

$$X(n) \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & \text{else} \end{cases}$$

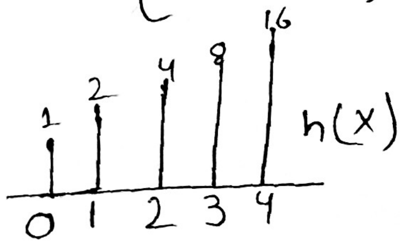
$$h(n) = \begin{cases} 2^n, & 0 \le n \le 4 \\ 0 & \text{else} \end{cases}$$

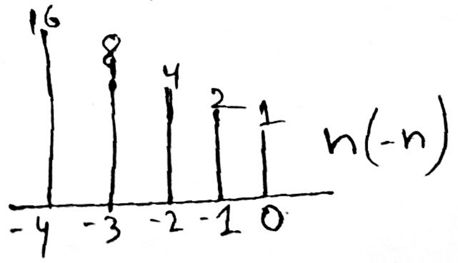
Solution:

$$x(n) = \{\alpha^{-2}, \alpha^{-1}, 1, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}\}$$



$$h(x) = \{1, 2, 4, 8, 16\}$$





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$\Rightarrow y[-3] = (\alpha^{-2})(1)$

$y[-3] = \alpha^{-2}$

$y[-2] = (\alpha^{-1})(1) + (\alpha^{-2})(2)$

$y[-2] = 2^{-1} + \alpha^{-2}$

$y[-2] = 2^{-3}$

$y[-1] = (2^{-1})(1) + (2)(4) + (2^{-1})(4) + (2^{-2})(8)$

$y[-1] = (1 \times 10) + 2^{-1} + 4\alpha^{-2}$

$y[-1] = 16\alpha^{-3} + 1$

$y[0] = (8)(\alpha^{-2}) + (\alpha^{-1})(4) + (1)(2) + (\alpha)(1)$

$y[0] = 8\alpha^{-2} + 4\alpha^{-1} + 2 + \alpha$

$y[0] = 12\alpha^{-3} + 2 + 2$

$\Rightarrow y[1] = (2^{-1})(8) + (1)(4) + (\alpha)(2) + (\alpha^{2})(1)$

$= 8\alpha^{-1} + 4 + 2\alpha + 2^{2}$

$y[2] = (\alpha^{-1})(16) + (1)(8) + (\alpha)(4) + (\alpha^{2})(2) + (\alpha^{3})(1)$

$y[2] = 16\alpha^{-1} + 8 + 4\alpha + 2\alpha^{2} + \alpha^{3}$

$y[3] = (1)(16) + (\alpha)(8) + (2^2)(4) + (\alpha^3)(2) + (\alpha^4)(1)$

$= 16 + 8\alpha + 4\alpha^2 + 2\alpha^3 + \alpha^4$

$y[4] = (\alpha)(16) + (\alpha^2)(8) + (\alpha^3)(4) + (\alpha^4)(2) + (\alpha^5)(1)$

$y[4] = 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5$

$y[5] = (\alpha^2)(16) + (2^3 8) + (\alpha^4)(4) + (\alpha^5)(2) + (\alpha^6)(1)$

$y[5] = 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \cancel{2}\alpha^6$

$y[6] = (\alpha^3)(16) + (\alpha^4)(8) + (\alpha^5)(4) + (\alpha^6 \times 2)$

$y[6] = 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$

$y[7] = (16)(\alpha^4) + (\alpha^5)(8) + (\alpha^6)(4)$

$y[7] = 16\alpha^4 + 8\alpha^5 + 4\alpha^6$

$y[8] = (\alpha^5)(16) + (\alpha^6)(8)$

$y[8] = 16\alpha^5 + 8\alpha^2$

$y[9] = 16\alpha^6$

$y[10] = 0$

$\Rightarrow$ There is no overlap in $y[10]$

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Question no 3 :

Determin the z- transform of the following signals and also sketch its Region of Convergence (ROC).

(i) $x(n) = \begin{cases} (\frac{1}{4})^n , & n \geq 0 \\ (\frac{1}{3})^{-n} , & n < 0 \end{cases}$

Solution:

$$x(n) = \begin{cases} (\frac{1}{4})^n , & n \geq 0 \\ (\frac{1}{3})^{-n} , & n < 0 \end{cases}$$

writing in the form of z-transform.

$$X(z) = \sum_{n=0}^{\infty} (\frac{1}{4})^{n} z^{-n} + \sum_{n=-\infty} (\frac{1}{3})^n z^n - 1$$

using geomatric series.

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} (\frac{1}{3})^n z^n - 1$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}} - 1$$

$$= \frac{1-\frac{1}{\frac{1}{4}}z^{-1} + 1-\frac{1}{1}-1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z^{-1})} - 1$$

$$= \frac{1-\frac{1}{3} + 1-\frac{1}{4}z^{-1} - (1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

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$$= \frac{1 - \frac{1}{3}z + 1 - \frac{1}{4}z^{-1}\left(1 + \frac{1}{3}z - \frac{1}{4}z^{-1} + \frac{1}{12}\cancel{z}\cdot\cancel{z}\right.}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{3}z\right)}$$

$$= \frac{1 - \cancel{\frac{1}{3}}z + 1 - \frac{1}{4}\cancel{z} \cdot 1 + \cancel{\frac{1}{12}} + \cancel{\frac{1}{4}} + \frac{1}{12}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{3}z\right)}$$

$$= \frac{1 + \frac{1}{12}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{3}z\right)}$$

$$= \frac{\frac{13}{12}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{3}z\right)}$$

Hence The Roc is $\frac{1}{4} < |z| < 3$.

(ii) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

Solution:

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^4 - 3^n, & n \geq 0 \\ 0, & \text{elsewhere.} \end{cases}$$

In the form of z-transform.

$$x(z) = \sum_{A=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 0\,3^n z^{-n}$$

using geomatric series to simplyfie it.

$$= \frac{1}{1 - \frac{1}{2}z^{?}} - \frac{1}{1 - 3z^{-1}}$$

$$= \frac{\cancel{1} - 3z^{-1} - \cancel{1} + \frac{1}{2}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - 3z^{-1}\right)}$$

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$$= \frac{-\frac{5}{2} z^{-1}}{(1-\frac{1}{2} z^{-1})(1-3 z^{-1}}$$

The ROC is $|z|73$.

——— x ——— x ——— x ——— x ———