ID = 13132 (1) Ahmad Zaib

question no (1)

=> part (A)

Determine the response $y(n), n \geq 0$, of the system describe by the second order difference equation.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

To the input $x(n) = (-1)^n u(n)$ and the initial condition are $y(-1) = y(-2) = 0$.

Solution:- $\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$ Hence

$$yh(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is

$$yp(n) = K(-1)^n u(n)$$

subsituting this solution into the difference equation we obtain:

$$\Rightarrow K[-1]^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

$\Rightarrow$ For $n=2$ $K(1+4+4)=2$

$K=2/9$ the total solution is

$y(n)-[(1)^n+(2n)^n+2/9(-1)^n]u(n)$

From the initial condition

we obtain, $y(0)=1, y(1)=2$

than

$$c_1+\frac{2}{9}=1$$

$$\Rightarrow c_1=7/9$$

$$\Rightarrow 2c_1+2c_2-2/9=2$$

$$\Rightarrow c_2=1/3$$

$\longleftrightarrow$

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Question (no1)

→ part (B)

Determine the impulse response and unit step response of systems described by the difference equation.

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

Solution:-

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = \frac{1}{2}, \frac{1}{5} \quad \text{Hence}$$

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$$

with $x(n) = \delta(n)$ we have

$$y(0) = 2$$

$$y(1) - 0.7\, y(0) = 0$$

$$\Rightarrow y(1) = 1.4$$

Hence, $c_1 + c_2 = 2$

And,

$$1/2\; c_1 + 1/5 = 1.4$$

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$1.4 = 7/5$

$\Rightarrow C_1 + \frac{2}{5} C_2 = 14/5$

these equations yield

$C_1 = 10/3 \; , \; C_2 = -4/3$

$h(n) = [10/3 (1/2)^n - 4/3 (1/5)^n] u(n)$

the step response is

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$s(n) = 10/3 \sum_{k=0}^{n} (1/2)^{n-k} - 4/3 \sum_{k=0}^{n} (1/5)^{n-k}$$

$$s(n) = 10/3 (1/2)^n \sum_{k=0}^{n} 2^k - 4/3 (1/5)^n \sum_{k=0}^{n} 5^k$$

$$s(n) = \frac{10}{3} (1/2)^n (2^{n+1} - 1) u(n) - 1/3$$

$$(1/5)^n (5^{n+1} - 1) u(n).$$

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Question no (2)

→ part (A):

Determine the causal signal $x(n)$ having the Z-transform.

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Solution:-

Taking inverse and Z-transfo

$$\frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{(z^{-1})}{(1-z^{-1})^2}$$

$A=4$, $B=-3$, $C=-1$

Hence

$$x(n) = [4(2)^n - 3 - n]\,u(n)$$

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Question no2:-

=> Part (B):

perform the circular convolution of the following two sequences. solve the problem step by step.

$$x_1(n) = [\underset{\uparrow}{2}, 1, 2, 1]$$

$$x_2(n) = [\underset{\uparrow}{1}, 2, 3, 4]$$

Solution:

each sequence consist of four nonzero points for the purpose of illustrating the operations involved in circular convolution it is desired to graph each sequence as points on a circle thus the sequence $x_1(n)$ and $x_2(n)$ are graphed as illustrated we note that the

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sequences are graphed in a counterclock wise direction on a circle.

Now, $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with

$\Rightarrow$ $x_2(n)$ as specified Begning with $m=0$ we have. ~~$x_3(m)$~~

$$x_3(m) = \sum_{n=0}^{3} x_1(n)\, x_2[(-n)]_N$$

$x_2(-n)_4$ is simply the sequence $x_2(n)$ folded and graphed on a circle the product sequence is obtained by multiplying $x_1(n)$ with $x_2(-n)_4$ point by point finally we sum the values in the product sequence to obtain

$$x_3(0) = 14$$

for $m=1$, we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n)_4$$

it is easily verified that $x_2(1-n)_4$ is simply the sequence $x_2(-n)_4$ rotated counter clockwise by one unit in the time.

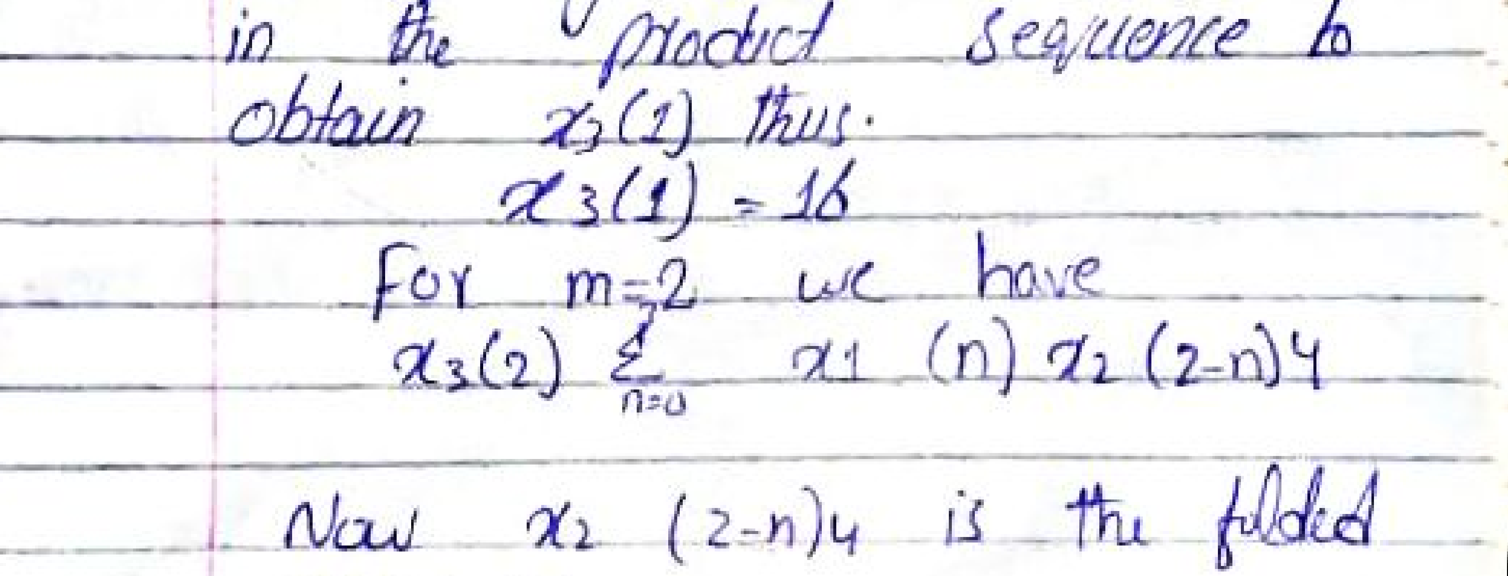
This rotated sequence multiplies $x_1(n)$ to yield the product sequence also finally we sum the values in the product sequence to obtain $x_3(1)$ thus.

$$x_3(1) = 16$$

For $m = 2$ we have

$$x_3(2) \sum_{n=0}^{3} x_1(n)\, x_2(2-n)_4$$

Now $x_2(2-n)_4$ is the folded sequence:



(A)

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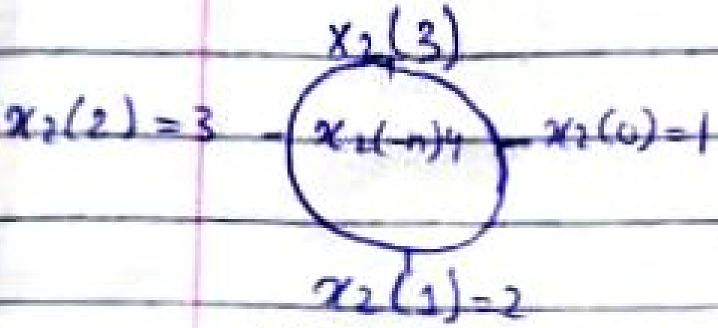
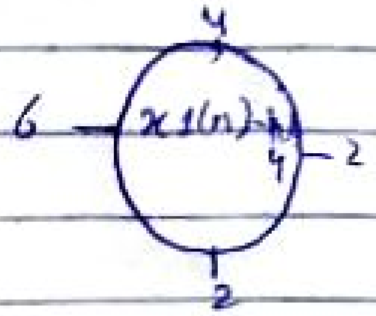
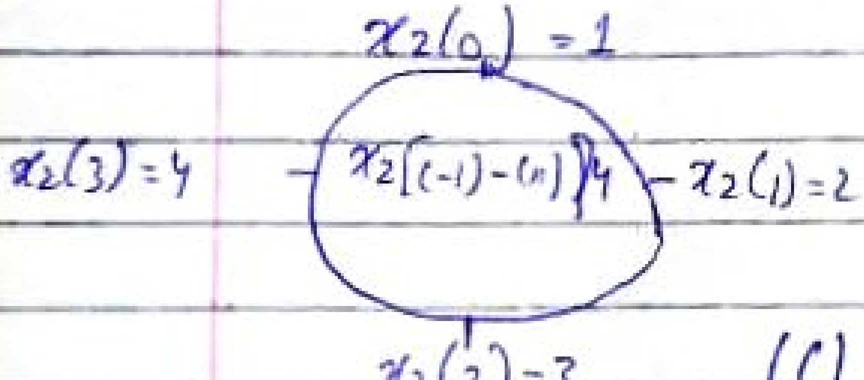
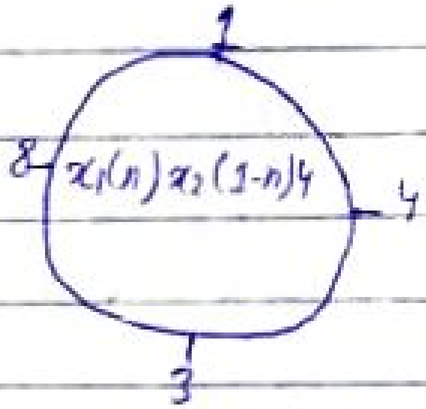




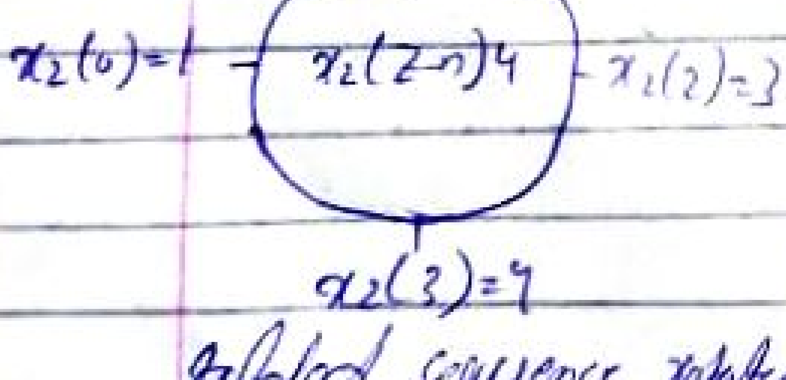
Folded sequence (B) product sequence

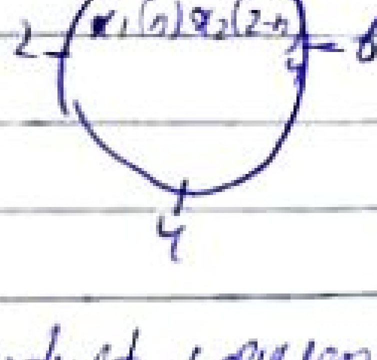




(C)

folded sequence rotated by one unit in time product sequence





(D)

folded sequence rotated by two unit in time product sequence

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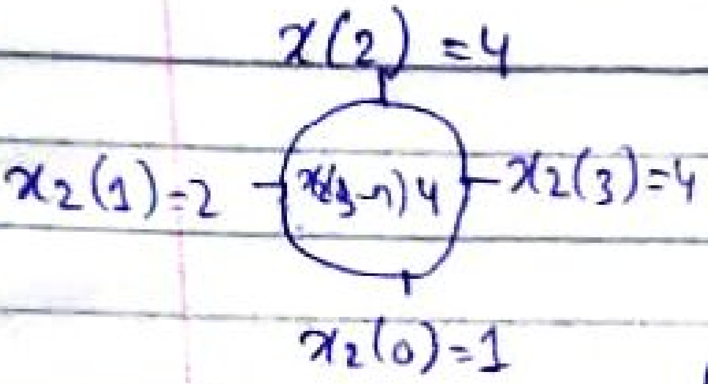
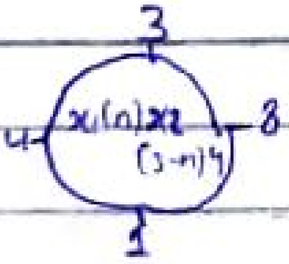




(E)

=> Folded sequence rotated by three unit in time

=> product sequence.

Q3(B)

Solution :

Give Data :

$\omega = \pi/2$

$\omega = 0$ and $\omega = \pi$

$1/\sqrt{2}$ at $\omega = 4\pi/9$

By The filter requirement:

Poles $= p_{1,2} = re^{\pm j\pi/2}$

Zeroes $= z_{1,2} = \pm 1$

$$\Rightarrow H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

$$= G = \frac{2}{-1+r^2}$$

$$= G = \frac{1-r^2}{2}$$

To set r use $H\frac{4\pi}{9} = 1/\sqrt{2}$

requirement.

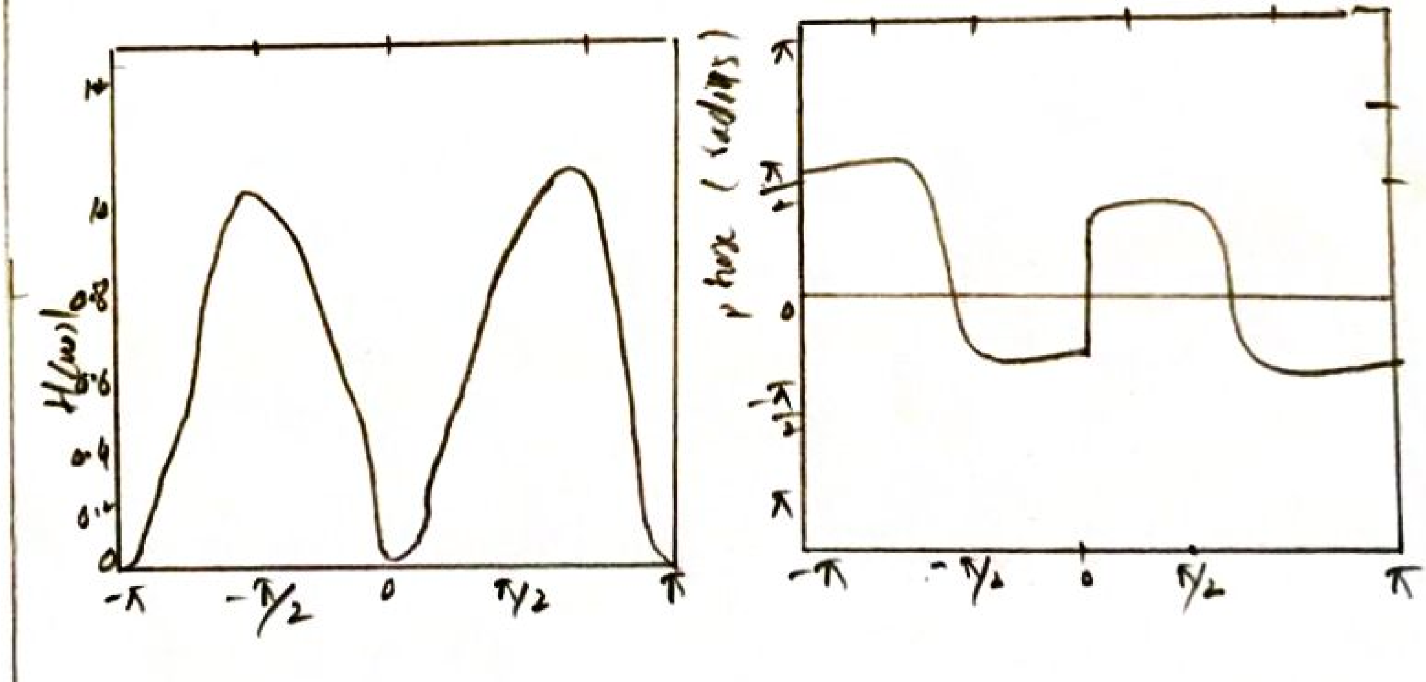
Now

$$|H(\tfrac{4\pi}{9})|^2 = \frac{(1-r^2)^2}{4}\,\frac{2-2\cos(8\pi/9)}{1+r^4-2r^2\cos(8\pi/9)}$$

$$= 1/2$$

Evaluating gives $r^2 = 0.7$ Therefore

$$H(z) = 0.15\,\frac{1-z^{-2}}{1+0.7z^{-2}}$$



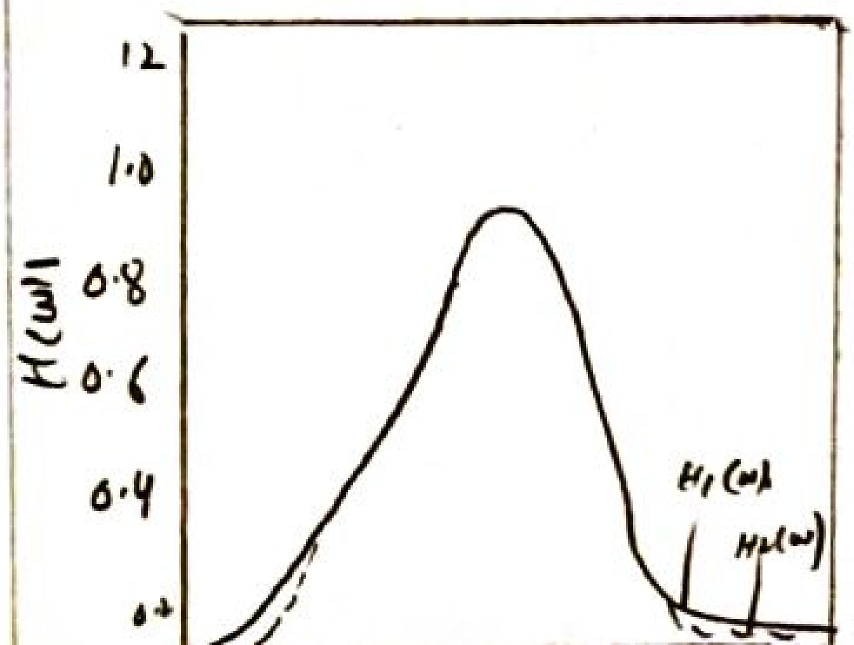
Q3 (a) A two pole low pass filter has the system response

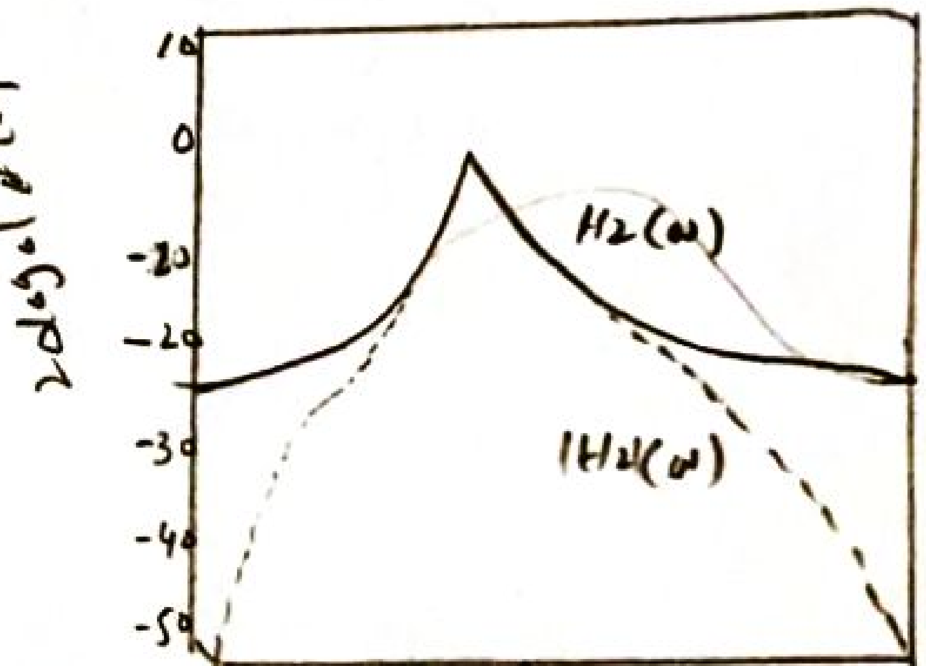
$$H(z) = \frac{b_0}{(1 - pz^{-1})^2}$$

$$H(0) = 1 \text{ and } \left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{1}{2}$$

Solution:

A two lowpass filter





Now The values of b_0 and p

$$H(0) = 1$$

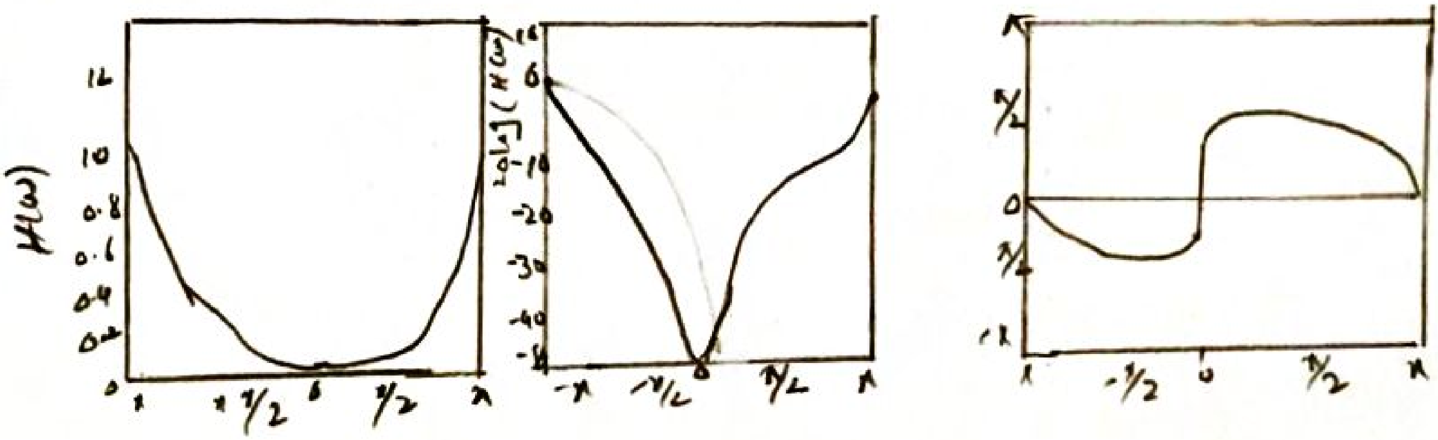
$$|H(\pi/4)|^2 = \frac{1}{2}$$

At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence

$$b_0 = (1-p)^2$$



At $= \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{1-pe^{j\pi(4)/2}}$$

$$= \frac{1-p^2}{1-p\cos(\pi/4)+jp\sin(\pi/4)^2}$$

$$= \frac{(1-p)^2}{(1-p\sqrt{2}+jp/\sqrt{2})^2}$$

Hence

$$\frac{(1-p)^4}{[(1-p/\sqrt{2})^2 + p^2/2]^2} = \frac{1}{2}$$

or Equivalently

$$\sqrt{2}(1-p)^2 = 1 + p^2 - \sqrt{2}p$$

The value of $p = 0.32$ Satisfies the Equation. Consequenty. The System function for The desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$