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Course : Linear Algebra

BS : CS

Examination : Final paper (online)

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Q.NO. 1:

Solution:

$$x_1 - 1x_1 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

Sol in matrix form

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 \\ 8 \\ 10 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{vmatrix}$$

$$|A| = 1(-10+0) + 1(0+40) + 1(0-10)$$
$$= -10 + 40 - 10 = 20$$
$$|A| = 20$$

Solution: Possible

Consider Augumented matrix $[A|b]$

$$[A|b] \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -2 & 8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right] R_3 - 5R_1$$

$$R \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -2 & 8 & 8 \\ 0 & 5 & -10 & 10 \end{array}\right] \frac{R_3}{5} \; \& \; \frac{R_2}{-2}$$

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$$R \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & +1 & -4 & -4 \\ 0 & 1 & 2 & 2 \end{array}\right] R_3 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -4 & -4 \\ 0 & 0 & 2 & 6 \end{array}\right] \frac{R_3}{2}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -4 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

from R_3 we have

$\boxed{x_3 = 3}$

from R_2 we have

$x_2 - 4x_3 = -4$

$x_2 - 4(3) = -4$

$\boxed{x_2 = 8}$

from R_1

$x_1 - x_2 + x_3 = 0$

$x_1 - 8 + 3 = 0$

$x_1 - 5 = 0$

$x_1 = 5$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 3 \end{pmatrix}$$ Req Consistant solution of the system

Q.No.2.

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4^{th} ID \\ 5 & -2 & 7 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 9 \\ 5 & -2 & 7 \end{bmatrix}$$

Find co-factor of every element of A

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A_{11} = \begin{vmatrix} -1 & 9 \\ -2 & 7 \end{vmatrix} = -7+18 = 1$$

$$A_{12} = \begin{vmatrix} 2 & 9 \\ 5 & 7 \end{vmatrix} = 14-45 = -31$$

$$A_{13} = \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = -4+5 = 1$$

$$A_{21} = \begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = 28+10 = 38$$

$$A_{22} = \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = 21 - 25 = -4$$

$$A_{23} = \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -6 - 20 = -26$$

$$A_{31} = \begin{vmatrix} 4 & 5 \\ -1 & 9 \end{vmatrix} = 36 + 5 = 41$$

$$A_{32} = \begin{vmatrix} 3 & 5 \\ 2 & 9 \end{vmatrix} = 27 - 10 = 17$$

$$A_{33} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -3 - 8 = -11$$

$$A = \begin{bmatrix} 11 & 31 & 1 \\ -38 & -4 & 26 \\ 41 & -17 & -11 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 11 & -18 & 41 \\ 31 & -4 & -17 \\ 1 & 26 & -11 \end{bmatrix}$$

$$|A| = a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$$

$$|A| = 3 \times 11 + 4 \times 31 + 5 \times 1$$

$$|A| = 33 + 124 + 5$$

$$|A| = 162$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A$$

$$A^{-1} = \frac{1}{162} \begin{bmatrix} 11 & -38 & 41 \\ 31 & -4 & -17 \\ 1 & 26 & -11 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{11}{162} & \frac{-38}{162} & 41/162 \\ 31/162 & -4/162 & 17/168 \\ 1/162 & 26/162 & -11/162 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 11/162 & -19/162 & 41/162 \\ 31/162 & -2/81 & -17/162 \\ 1/162 & 13/81 & -11/162 \end{bmatrix}$$

<u>Answer</u>

Q.No.3

Solution:

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Converting given equation into matrix form.

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$R_1 \leftarrow R_1 \div 2$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$R_2 \leftarrow R_2 - R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$R_3 \leftarrow R_3 - 3 \times R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -9 & -13 \end{array}\right]$$

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$R_2 \leftarrow R_2 \div 2$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right)$$

$R_1 \leftarrow R_1 - R_2$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right)$$

$R_3 \leftarrow R_3 + R_2$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -9 & -11 \end{array}\right)$$

$R_3 \leftarrow R_3 \div -9$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & \frac{11}{9} \end{array}\right)$$

$R_1 \leftarrow R_1 - 2 \times R_3$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{41}{9} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & \frac{11}{9} \end{array}\right)$$

i.e.

$$x = \frac{41}{9}$$

$$y = 2$$

$$z = \frac{11}{9}$$

Solution By Gauss jordan method

$x = \frac{41}{9}$, $y = 2$ and $z = \frac{11}{9}$

Date:

Q-No.4:

Solution:

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

First, the find eigenvalues and eigenvectors.

Eigenvalue: 5, eigenvector: $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Eigenvalue: 2, eigenvector: $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{bmatrix}$

eigenvalue: 1, eigenvector: $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \\ 1 \end{bmatrix}$

Form the matrix P.

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{4} \\ 1 & 1 & 1 \end{bmatrix}$$

Form the diagonal matrix D.

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

INFINITY

These matrices have the property that $A = PDP^{-1}$

$$A = PDP^{-1}$$

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{4} \\ 1 & 1 & 1 \end{bmatrix}$$

Decimal form:

$$P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 1 & 0.5 & 0.25 \\ 1 & 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q.No.5.

Solution :

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 25x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

In matrix form

$$\begin{bmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since the system has no trivial sols

$$|A| = 0$$

Now consider Ausumented matrix $[A|0]$ For non-trivial Solution.

$$[A|0] = \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -25 & 4 & 0 \\ 0 & 1 & -8 & 0 \end{array}\right] \begin{array}{l} R_2 + R_1 \\ R_3 - 2R_1 \end{array}$$

$$R \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right] \frac{R_2}{-20} \text{ \& } \frac{R_3}{-9}$$

$$R \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] R_3 - R_2$$

INFINITY

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

from R_2 we have

$$\boxed{x_2 = 0}$$

from R_1 we write

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$3x_1 + 0 - 4x_3 = 0$$

$$3x_1 - 4x_3 = 0$$
$$3x_1 = 4x_3$$
$$x_1 = \frac{4}{3}x_3$$

let $x_3 = r$ Conjunct no

So $x_1 = \frac{4}{3}r$

So the solution vector of the system $Ax = 0$ is

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{4}{3}r \\ 0 \\ r \end{pmatrix}$$

INFINITY

Q.No.6

Solution:

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

Rank $= 2$

To make unit matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to 2R_3 - R_2$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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Rank of matrix by normal form

Column operation

$C_2 \rightarrow C_2 - 3C_1$
$C_3 \rightarrow C_3 - 4C_1$
$C_4 \rightarrow C_4 - 3C$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Swip column 4 with 2

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide Column 2 by -6

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Normal form matrix