Name Abdullah Jan

Id 16105

Semester 1^{st} summer

Section A

Q1 $\int_0^1 \frac{4t^3-2t^2+3t-1}{2t^2+1} dt$

Solution

$$= \int_0^1 \frac{4t^3-2t^2+3t-1}{2t^2+1} dt$$

$$= \int_0^1 \frac{t(4t^2+3)-(2t^2+1)}{2t^2+1} dt$$

$$= \int_0^1 \frac{t(4t^2+3)}{2t^2+1} dt - \int_0^1 \frac{\cancel{2t^2+1}}{\cancel{2t^2+1}} dt$$

$$= \int_0^1 \frac{t(4t^2+3)}{2t^2+1} dt - \int_0^1 1 dt$$

$$= \int_0^1 \frac{t(4t^2+3)}{2t^2+1} dt - t \Big|_0^1$$

$$= \int_0^1 \frac{t(4t^2+3)}{2t^2+1}\,dt - [1-0]$$

$$= \int_0^1 \frac{t(4t^2+3)}{2t^2+1} - 1 \quad \text{—①}$$

Now

let

$2t^2+1 = y \implies$

$$2t^2+1 = y$$
$$2t^2 = y-1$$
$$4t^2 = 2y-2$$
$$4t^2+3 = 2y-2+3$$
$$4t^2+3 = 2y+1$$

As

$t = 1$ ie $y = 3$

$t = 0$ ie $y = 1$

Now Diff

$$4t = \frac{dy}{dt}$$

$$dt = \frac{dy}{4t}$$

$$① \Rightarrow = \int_1^3 \frac{t(2y+1)}{y} \cdot \frac{dy}{dt} - 1$$

$$= \int_1^3 \frac{2y+1}{4y}\,dy - 1$$

$$= \frac{1}{4}\left[\int_1^3 \frac{2y\,dy}{y} + \int_1^3 \frac{1}{y}\,dy\right] - 1$$

$$= \frac{1}{4}\left[2y \Big|_1^3 + \ln y \Big|_1^3\right] - 1$$

$$= \frac{1}{4}\left[2(3) - 2(1) + \ln(3) - \ln(1)\right]$$

$$= \frac{1}{4}\left(6 - 2 + 1.0986\right] - 1$$

$$= \frac{1}{4}\left[5.0986\right] - 1$$

$$= 1.27465 - 1$$

$$= 0.2746 \quad \underline{\text{Ans}}.$$

2 $\int_2^3 t \sin t^2 \, dt$

Solution

let $t^2 = u$

$$2t\,dt = du$$

$$t\,dt = \frac{du}{2}$$

when $t \to 2$ Then $u \to 4$

when $t \to 3$ Then $u \to 9$

$$\int_2^3 t \sin t^2 dt = \int_4^9 \sin u \frac{du}{2}$$

$$= 2 \int_4^9 \sin u\, du$$

$$= 2 \left| (-\cos u) \right|_4^9$$

$$= 2 \{ -\cos(9) + \cos(4) \}$$

$$= 2 \{ \cos(4) - \cos(9) \}$$

$$= 2 \times \{ 0.9975 - 0.9876 \}$$

$$\int_2^3 t \sin t^2 dt = 0.0198 \quad \underline{\text{Ans}}.$$

Scanned with CamScanner