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Assigment # Differentional Equation

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Cauchy Eular Method

Q# 1

$$x^3 y''' + 2x^2 y' + 2y = 10x + \frac{10}{x}$$

Sol:-

$$x^3 y''' + 2x^2 y' + 2y = 10x + \frac{10}{x} \rightarrow ①$$

Put $x = e^t$ then

$$\frac{dx}{dt} = e^t \Rightarrow \frac{dt}{dx} = e^{-t}$$

Now

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dt} \cdot e^{-t}$$

OR

$$\frac{dy}{dx} = \frac{dy}{dt}$$

$$y' = \frac{dy}{dx} = e^{-t} Dy \quad \because \frac{d}{dt} \rightarrow D$$

Similarly

$$y'' = e^{-2t} [D(D-1)]$$

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$$y''' = e^{-3t}\,[D(D-1)(D-2)]$$

Using these Values in ①

$$e^{3t} \cdot e^{-3t}\,[D(D-1)(D-2)]\,y + 2e^{2t} \cdot e^{-2t}\,[D(D-1)]\,y$$

$$+2y = 10e^{t} + 10e^{-t}$$

$$\Rightarrow (D^3 - 3D^2 + 2D)\,y + (2D^2 - \cancel{2D})\,y + 2y$$
$$= 10e^{t} + 10e^{-t}$$

$$\Rightarrow D^3y - D^2y + 2y = 10e^{t} + 10e^{-t}$$

$$\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2} + 2y = 10e^{t} + 10e^{-t} \longrightarrow ②$$

the assosialated homogenious Equation

$$\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2} + 2y = 0$$

Say, $\frac{d}{dt^2} = k^2$, $\frac{d^3}{dy^3} = k^3$

$$\Rightarrow (k^3y - k^2y + 2y) = 0$$

$$\Rightarrow (k^3 - k^2 + 2)\,y = 0$$

For non-trivial Sol-, $y \neq 0 \Rightarrow$

$$k^3 - k^2 + 2 = 0$$

Roots are $k = -1,\ 1 \pm i$

$\Rightarrow y_c(t) = Ae^{-t} + (B\cos t + C\sin t)e^t$

Where is Complumentry S-l

$\longleftrightarrow$

Q No:- 3

$$x^2y'' + 2xy' - 6y = 10x^2 = y(1) = 1$$

$$y'(1) = -6$$

Sol:

$$x^2y'' + 2xy' - 6y = 10x^2 \rightarrow ①$$

$$y(1) = 1$$

$$y'(1) = -6$$

Let

$x = e^t$ i.e $t = \log x$

Now

$$xy' = \Delta y \Rightarrow x^2y' = \Delta(\Delta - 1)y$$

Where $\Delta = \frac{d}{dt}$

then equation ① $\Rightarrow [\Delta(\Delta-1)y +$
$2\Delta - 6]y = 10e^{2t}$

$[\Delta^2 - \Delta + 2\Delta - 6]y = 10e^{2t}$

$\Rightarrow [\Delta^2 + \Delta - 6]y = 10e^{2t}$

Char from Equation $\Delta^2 + \Delta - 6 = 0$

$\Delta + 3\Delta - 2\Delta - 6 = 0$

$\Delta = -3, \; \Delta = 2$

Complementary function

$C.F = C_1 e^{-3t} + C_2 e^{2t}$

Also P. Integral

$P.I = \frac{1}{\Delta^2 + \Delta - 6} 10e^{2t}$

$= 10 \frac{1}{(2)^2 + 2 - 6} e^{2t}$ replace Δ by 2

Case of failure

$P.I = 10t \frac{1}{2\Delta + 1} e^{2t} = 10t \cdot \frac{1}{2(2) + 1} e^{2t}$

$= 10t \; \frac{1}{5} e^{2t} = 2t \, e^{2t}$

Hence general Sol = C.f + P.I

$y = C_1 e^{-3t} + C_2 e^{2t} + 2t \, e^{2t}$

$C_1 x^{-3} + C_2 x^2 + 2 (\log x) \, x^2$

Apply Initial Condition

$y(1) = 1$ we get

$1 = C_1 + C_2 + 0 \rightarrow (A)$

& $\dot{y}(1) = -6$

$y' = -3 C_1 x^{-4} + 2 C_2 x + 2x + 4x \log x$

$-6 = 3C_1 + 2C_2 + 2 + 0$

$-3C_1 + 2C_2 = -8 \rightarrow (B)$

eq (A) x3 and add with (B)

$3 = \cancel{3C_1} + 3C_2$
$-8 = -\cancel{3C_1} + 2C_2$

$5C_2 = -5$

$\boxed{C_2 = -1}$

eq Ⓐ $\Rightarrow 1 - C_1 - 1$

$\boxed{C_1 = 2}$

thus ☆ $\Rightarrow$ $\boxed{y = 3x^{-3} - x^2 + 2x^2 \log x}$

$\longleftrightarrow$

Question # 64)

$x^2 y'' + 7x y' + 5y = x^3$;

$y(0) = 2$ & $y'(1) = 2$

Sol:-

let

$x = e^t \Rightarrow t = \log x$, $\Delta = \frac{d}{dt}$

Now $xy' = \Delta y \Rightarrow x^2 y' = \Delta(\Delta - 1) y$

then

$(\Delta(\Delta-1) + 7\Delta + 5)y = e^{5t}$

$(\Delta^2 + 6\Delta + 5)y = e^{5t}$

Char eq is $\Delta^2 + 6\Delta + 5 = 0$

$\Delta^2 + 5\Delta + \Delta + 5 = 0$

$\Delta = -5, -1$

Complementare eq is

$C.F = C_1 e^{-5t} + C_2 e^{-t}$

$\Rightarrow$ P. Integral

$$P.I = \frac{1}{\Delta^2 + 6\Delta + 5} e^{5t}$$

$$= \frac{1}{5^2 + 6(5) + 5} \quad \text{replacing } \Delta \text{ by } 5$$

$$= \frac{1}{60} e^{5t}$$

Thus

$$y = C_1 e^{-5t} + C_2 e^{-t} + \frac{1}{60} e^{5t}$$

$$y = C_1 x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5$$

$$y' = -5C_1 x^{-6} - C_2 x^{-2} + \frac{1}{12} x^4$$

$y(0) = 2 \qquad x = 0, \; y = 2$

$$2 = C_1 + C_2 + \frac{1}{60}$$

$C_1 + C_2 = 119/60 \longrightarrow$ (a)

$y'(1) = 2 \qquad x = 1, \; y' = 2$

$$2 = -5C_1 - C_2 + 1/12$$

$-5c_1 - C_2 = 23/12 \longrightarrow$ B

$A + B \quad -4c_1 = \frac{234}{60} \Rightarrow 4 = \frac{-117}{120}$

Now

$$y = \frac{-117}{120} x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5$$

$C_1 = \frac{-117}{120}$ put in eq A $\frac{-117}{120}$

$+ C_2 = \frac{119}{60}$

$C_2 = \frac{119}{60} + \frac{117}{120}$

$$\boxed{= \frac{238 + 117}{120} = \frac{355}{120}}$$

$\longleftrightarrow$

Question #(02)

$$x^3 y''' + 4x^2 y''' - 5xy' - 15y = x^4$$

Sol:-

let $x^3 y''' + 4x^2 y''' - 5xy' - 15y = x^4 \rightarrow (1)$

$x^2 y = e^t \Rightarrow x = e^t - 1$

Diff $\log(x.y) = t$

Also

$(xy) = \Delta y \left\{ \frac{d}{dt} = \Delta \text{ and } D = \frac{d}{dx} \right\}$

then eq (1)

$$(\Delta(\Delta-1) - 15\Delta + 5)y = (e^t - 1)^2$$

$$(\Delta^2 - 15\Delta + 5)y = e^{2t} - 2e^t + 1$$

Char eq is $(\Delta^2 - 15\Delta + 5)y = 0$

$$(\Delta - 5)^2 = 0$$

$$\Delta = 5, 5$$

the Complementry function is

$$C.F = (C_1 + C_2 t)\, e^{2t}$$

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Also partial Integral is

$$P.I = \frac{1}{(\Delta-5)^2}\left(e^{2t}-2e^{t}+1\right)$$

$$= \frac{1}{(\Delta-2)^2} e^{2t} - 5 \frac{1}{(\Delta-5)^2} e^{t} + \frac{1}{(\Delta-5)^2} \rightarrow (2)$$

Now $\frac{1}{(\Delta-5)^2} e^{2t} = \frac{1}{(5-5)^2} e^{2t} = \frac{1}{0} e^{2t}$

Case of failure

$$\frac{1}{(\Delta-5)^2} e^{2t} = t \frac{1}{5(1-5)^2} e^{t} = t^2 \frac{e^t}{5}$$

and $\frac{1}{(\Delta-5)^2} \cancel{e^{2t}}(1) = 5 \frac{1}{(1-5)^2} e^{t} = 5e^t$

and

$$\frac{1}{(\Delta-5)^2}(1) = \frac{1}{(\Delta-5)^2} e^{\cdot t} = \frac{1}{15}$$

eq (2) $\Rightarrow PI = \frac{1}{5} t^2 e^{3t} - 5e^{t} + \frac{1}{15}$

Hence Complete Saloution is

$\Rightarrow y = C.F + P.I$

$$y = (C_1 + C_2') e^{2t} + \frac{1}{5} t^2 e^{2t} - 5e^{t} + \frac{1}{15}$$

$y = C.F + P.I$

$\Rightarrow$

$$y = (c_1 + c_2 t)\, e^{2t} + \frac{1}{5} t^2 e^{2t} - 5e^t + \frac{1}{15}$$

Repeat Value of e^t

$$y = (c_1 + c_2 \log (x+1))(x+1)^2 + \frac{1}{2}$$
$$\left[(\log (x+1)^2)\,(x+1)^2\right] - 5(x+1)$$
$$+ \frac{1}{15}$$

$\longleftrightarrow$

Question #(5)

$$(x+1)^2 y'' - 3(x+1) y' + 4y = x^2 = ①$$

Sol:

$$(x+1)^2 y'' - 3(x+1) y' + 4y = x^2 \rightarrow ①$$

let

$$x+1 = e^t \Rightarrow x = e^t - 1$$

Diff $\log (x+1) = t$

Also

$$(x+1)\, y' = \Delta y, \left[\frac{d}{dt} = \Delta \text{ and } D = \frac{d}{dx}\right]$$

Case of failure

$$\frac{1}{(\Delta-2)^2} e^{2t} = t \frac{1}{2(1-2)^2} e^t = \frac{t^2 e^t}{2}.$$

and $2 \frac{1}{(\Delta-2)^2} e^t = 2 \frac{1}{(1-2)^2} e^t = 2e^t$

and $\frac{1}{(\Delta-2)^2} (1) = \frac{1}{(\Delta-2)^2} . e^{0t} = \frac{1}{4}$

eq ② $\Rightarrow$ $P.I = \frac{1}{2} t^2 e^{3t} - 2e^t + \frac{1}{4}$

Hence Complete Solution is

$$y = C.F + P.I$$

$$y = (C_1 + C_2') e^{2t} + \frac{1}{2} t^2 e^{2t} - 2e^t + \frac{1}{4}$$

Repect Value of e^t

$$y = C_1 + C_2 \log (x+1)(x+1)^2 + \frac{1}{2}\left(\log (x+1)^2 (x+1)^2\right] - 2x - 7/4.$$

Which is the required

$\longleftrightarrow$