FINAL TERM EXAM

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Question 1) Apply both Euler's method (1) and the improved Eulers method to the solution of

$$\frac{dy}{dx} = 2x, \quad y(0) = 1$$

For $0 \leq x \leq 0.5$ using $h = 0.1$. Compare your answer with the analytic sol. Work throughout to three decimal places.

Ans: By Euler's Method:

Given data:

$y = (0) = 1, \; h = 0.1 \quad x_0 = 0$

By formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h\,[2x_n]$$

"1st Iteration":

$n = 0$

$$y_1 = y_0 + h(2x_0)$$

$$y_1 = 1 + 0.1(2(0))$$

$$y_1 = 1 + 0.1$$

$$\boxed{y_1 = 1.0}$$

→ $x_{n+1} = x_n + h$

$$x_1 = x_0 + h$$

$$x_1 = 0 + 0.1$$

$$\boxed{x_1 = 0.1}$$

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2nd Iteration:

②

$$n = 1$$

$$y_2 = y_1 + h(2x_1)$$

$$y_2 = 1.1 + 0.1(2(0.1))$$

$$\boxed{y_2 = 1.02}$$

$$x_{n+1} = x_n + h$$

$$x_2 = x_1 + h$$

$$x_2 = 0.1 + 0.1$$

$$\boxed{x_2 = 0.2}$$

3rd Iteration:

$$n = 2$$

$$y_3 = y_2 + h(2x_2)$$

$$y_3 = 1.02 + 0.1(2(0.02))$$

$$\boxed{y_3 = 1.06}$$

$$x_{n+1} = x_n + h$$

$$x_3 = x_2 + 0.1$$

$$x_3 = 0.2 + 0.1$$

$$\boxed{x_3 = 0.3}$$

b) By Modified Euler Method ③

$$\frac{dy}{dx} = 2x$$

Given data

$y_0 = 1, x_0, 0, h = 0.1$

Formula: $y^*_{n+1} = y_n + h\,[f(x_n)]$

$y^*_{n+1} = y_n + h\,(2x_n)$ — ①

$y_{n+1} = y_n + h/2\,[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})]$

$= y_n + \frac{h}{2}\,[2x_n + 2x_n]$

$= y_n + \frac{h}{2}\,[4x_n]$

Ist Iteration

$n = 0$

$x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1$

$\boxed{x_1 = 0.1}$

$y_1 = y_0 + \frac{h}{2}(4x_0)$

$y_1 = 1 + \frac{0.1}{2}(4(0))$

$y_1 = 1$

2nd Iteration

$n = 1$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$

$y_2 = y_1 + \frac{h}{2}(4x_1)$

$y_2 = 1 + \frac{0.1}{2}(4(0.1))$

$\boxed{y_2 = 1.02}$

3rd Iteration

$n = 2$

$x_3 = x_2 + h$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

$y_3 = y_2 + \frac{h}{2}(4x_2)$

$= 1.02 + \frac{0.1}{2}(4(0.2))$

$\boxed{y_3 = 1.06}$

Question 2) use the fourth-order Runge Kutta method to obtain a solution of

$$\frac{dy}{dx} = x^2 + x - y$$

Subject to $y=0$ when $x=0$, for $0 \le x \le 0.6$ with $h=0.2$. Work throughout to four decimal places.

Given data: $y=0,\ x=0,\ h=0.2 \qquad 0 \le x \le 0.6$

$y_{n+1} = y_n + k$

- **1st Iteration:**

$h=0$

$y_1 = y_0 + k,\ k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$K_1 = hf(x_n, y_n)$

$K_1 = h(x_0^2 - x_0 - y_0)$

$K_1 = 0.2(0^2 - 0 - 0)$

$\boxed{K_1 = 0}$

$K_2 = hf(x_n + \frac{h}{2}, y_n + \frac{h}{2})$

$= 0.2f(x_0 + h/2, y_0 + h/2)$

$= 0.2f(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2})$

$= 0.2f(0.1, 0.1)$

$= 0.2(0.1^2 + \cancel{0.1} - \cancel{0.1})$

$\boxed{K_2 = 0.0020}$

$$k_3 = hf\left(x_n + \frac{h}{2},\ y_n + \frac{k_2}{2}\right)$$

$$= 0.2f\left(0+\frac{0.2}{2},\ \frac{0+0.002}{2}\right)$$

$$= 0.2f(0.1,\ 0.001)$$

$$= 0.2\ (0.1^2 + 0.1 - 0.001)$$

$$\boxed{K_3 = 0.0218}$$

$$k_4 = hf(x_n + h,\ y_n + k_3)$$

$$= 0.2f(0+0.2,\ 0+0.0218)$$

$$= 0.2f(0.2,\ 0.0218)$$

$$= 0.2\ (0.2^2 + 0.2 - 0.0218)$$

$$\boxed{K_4 = 0.0436}$$

$$K = \frac{1}{6}\left(0 + 2(0.002) + 2(0.0218) + 0.0436\right)$$

$$K = 0.0152$$

$$y_1 = 0 + 0.0152$$

$$\boxed{y_1 = 0.0152}$$

Question 3) A rocket is released and travels at a variable speed v. A motion sensor on the rocket measures this speed and the value is sampled by an on-board computer at 1 second intervals. The computer is required to calculate the distance travelled by the rocket and relay the value to a ground station at regular intervals. Records values of the measurement taken by the computer during the first 10 seconds of flight. Assuming that the computer uses the trapezium rule to estimate the distance travelled by the rocket, calculate the value that the computer will relay to the ground station after 10 seconds.

Time	0	1	2	3	4	5	6	7	8	9	10
Speed	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

Ans: Given data:

$$a = 0,\ b = 10,\ n = 10$$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Solution:

x	0	1	2	3	4	5	6	7	8	9	10
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

using formula

$$f(x)\,dx = h/2\,[f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) \cdots f(x_9)) + f(x_{10})]$$

$$= 1/2\,[10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 +) + 62.1]$$

$$= 412.9$$

Question 4) Estimate the values of the 8 following integral using simpson's Rule.

$$\int_2^3 \ln(x^3+1)\,dx$$

use 10 strips.

Solution: $n = 10$

$$h = \frac{3-2}{10} = 0.1$$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now using formula:

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4(f(x_1)+f(x_3)\dots\right] + 2\left[f(x_2\dots) + f(x_n)\right]$$

$$= \frac{0.1}{3}\left[0.693 + 4(0.846+1.162+1.476+1.777) + 2(1.003 + 1.320 + 1.628 + 1.922) + 2.062\right]$$

$$= \boxed{1.184}$$