1

Name: Niamat Ullah

Semester: ist Semester.

Term: Mid

Subject: Calculus NS-112

Q No (1) The function $g(t)$ is defined by $g(t) = 0$ $t < 0$

t^2 $0 \le t \le 3$

$2t + 3$ $3 < t \le 4$

12 $t > 4$

(a) State any point of discontinuity

(b) find if they exist

i) $\lim_{t \to 3} g$

Sol: To check possibility of the discontinuity of the function is at $t = 0$ & 4

→ First at $t = 0$

$g(t) = t^2$

$g(0) = 0^2 = 0$

For R.H.L

$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$

Apply limit

$= 1 + 0^2 + (g(0)) = \underline{1}$

For L.H.L

$\lim_{h \to 0} g(1-h) = gt+3$ P.T.O (P=1)

$= \lim_{h \to 0} g(1-h)+3$

$= \lim_{h \to 0} g-gh+3$

Apply limit

$= g-g(0)+3$

$= 5$

$R.H.L \neq L.H.L = g(t) = 5$

Now at $t=4$

$g(4) = g(4)+3$

$= 8+3$

$= 11$

For R.H.L

$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} g(1+h)+3$

$= \lim_{h \to 0} g+gh+3$

$= g+g(0)+3 \Rightarrow 5$

For L.H.L

$\lim g(1-h) = 12$

$g(4) = R.H.L \neq L.H.L$

point of discontinuity is at $t=4$

P.T.O

(b) Find, if they exist

i) $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.L $\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + gh$

Apply limit

$= 1 + 3^2 + g(3) = 16$

L.H.L $\lim_{h \to 3} g(1-h) = \lim_{h \to 3} gt + 3$

$= \lim_{h \to 3} g(1-h) + 3$

$= \lim_{h \to 3} 2 - 2h + 3$

Apply limit

$= g - g(3) + 3$

$= g - 6 + 3$

$= -1$

R.H.L $\neq$ L.H.L (do not exist Since L.H.L is -ve)

End.

(P=3)

Q No 2: $y(x) = x^2 + \sin x$

Sol: By Maclaurin's Series expension we have

$$f(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \cdots\cdots$$

Now $f(x) = y(x) = x^2 + \sin x$

$$f'(x) = 2x + \cos x$$
$$f''(x) = x - \sin x$$
$$f'''(x) = -\cos x$$

Thus $f(0) = (0)^2 + \sin(0) = 0$

$$f'(0) = 2(0) + \cos(0) = 1$$
$$f''(0) = 0 - \sin(0) = 0$$
$$f'''(0) = -\cos(0) = -1$$

Hence by Maclaurin's Expension

$$f(x) = y(x) = 0 + x(1) + \frac{x^2(0)}{2!} + \frac{x^3(-1)}{3!}$$

$$= 0 + x + 0 - \frac{x^3}{3!} + \cdots\cdots$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$$

is The required Maclaurins Expension.

$\boxed{P=4}$

Q3 (i) Find y'' given

$$1 + xy = x^2 + y^2$$

Sol

$$\Rightarrow \frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2)$$

$$\Rightarrow \frac{d}{dx}(1)\ \frac{d}{dx}(y) = \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2)$$

$$\Rightarrow 0 + x\frac{dy}{dx} + y\frac{dx}{dx} = 2x + 2y\frac{dx}{dx}$$

$$\Rightarrow x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\Rightarrow \frac{dy}{dx}(x-2y) = (2x-y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x-y}{x-2y} \quad \text{———} \ (1)$$

$$\Rightarrow y' = \frac{dy}{dx} = \frac{2x-y}{x-2y}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{2x-y}{x-2y}\right)$$

P.T.O

$p=5$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y)-(2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$\Rightarrow y'' = \frac{(x-2y)(2-y')-(2x-y)(1-2y')}{(x-2y)^2}$$

$$\Rightarrow y'' = \frac{(2x-xy'-4y+2yy')-(2x-4xy'-y+2yy')}{(x-2y)^2}$$

$$\Rightarrow y'' = \frac{4xy'-3y-xy'}{(x-2y)^2}$$

$$\Rightarrow y'' = \frac{4x\left(\frac{2x-y}{x-2y}\right)-3y-x\left(\frac{2x-y}{x-2y}\right)}{(x-2y)^2}$$

$$\Rightarrow y'' = \frac{4x(2x-y)-3y(x-2y)-x(2x-y)}{(x-2y)(x-2y)^2}$$

$$\boxed{\Rightarrow y'' = \frac{6x^2+6y^2-6xy}{(x-2y)^3}}$$ Ans

$\boxed{P=6}$

Q No (8) (ii) $y = x^3(1+x)^9 e^{6x}$

Find y' by using logarithmic differentiation

Sol: Taking log both side

$\ln y = 3 \ln x + 9 \ln(1+x) + 6x$ $\qquad \ln e = 1$

$$\frac{1}{y} y' = 3\frac{1}{x} + 9\left(\frac{1}{1+x}\right) + 6$$

$$\frac{y'}{y} = \frac{3}{x} + \frac{9}{1+x} + 6$$

$$y' = y\left(1\,\frac{3}{x} + \frac{9}{1+x} + 6\right)$$

$$\boxed{y' = x^3(1+x)^9 e^{6x}\left(\frac{3}{x} + \frac{9}{1+x} + 6\right)}$$

End

P=7